Drizzle Droplet Distribution Retrieved with Ka-band Vertically Pointing Radar

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***Abstract*— This study uses vertically pointing Doppler lidar and Ka-band Doppler radar to estimate drizzle droplet distributions falling below cloud base in maritime warm boundary-layer (WBL) clouds. The aerosol rich environment enabled the Doppler lidar to measure the vertical air motion, even in the presence of falling drizzle droplets. The Ka-band Doppler velocity power spectra observed the falling drizzle droplets, which are shifted upwards or downwards by the vertical air motion and broadened by turbulent broadening effects. The retrieval algorithm consists of four steps: (1) deconvolve the observed Ka-band Doppler velocity spectra to remove turbulent broadening, (2) shift the spectra by the observed Doppler lidar air motion, (3) convert reflectivity spectral density in each velocity bin to number concentration, and (4) convert each velocity bin to drizzle diameter so that the discrete drizzle droplet distribution is in units of number concentration per drizzle drop diameter.**

Keywords—drizzle droplet distribution; Doppler velocity spectra

I. Introduction

Rain formation in warm boundary-layer (WBL) clouds plays an important role in Earth's radiative balance and hydrological cycle, yet the associated microphysics and radiation processes are poorly represented in the world's leading Earth System Models (ESMs). When compared with satellite observations, ESM models show compensating biases relative to satellite observations. These model biases include precipitation that is "too frequent, too light" and tropical clouds that are "too few, too bright". We hypothesize that these model biases stem from common assumptions representing warm rain processes and that identifying and improving their structural errors will lead to improved atmospheric system predictability.

To improve rain representation in numerical models, it is advantageous to retrieve drizzle droplet distributions in the real atmosphere with as few assumptions as possible. For example, retrieving number concentration at discrete velocity bins without assuming a particular distribution shape (i.e., Gamma or log-normal distribution), will not bias the input datasets used to determine appropriate rain representations in numerical models.

This study uses observations collected on the Scripps Pier in San Diego, California, during the year-long US Department of Energy (DOE) EPCAPE field campaign. The key instruments used in this study include a ceilometer to determine cloud base, a vertically pointing Doppler lidar to measure vertical air motion from aerosol scattering, and a Ka-band zenith pointing radar (KAZR) to measure Doppler velocity power spectra of drizzle sized hydrometeors below the ceilometer cloud base.

II. Retreival Method

The recorded Doppler velocity power spectra (units of Watts) are converted to a reflectivity spectral density $S_{obs}(v)$ (units of $mm^6\ m^{-3}\ (m\ s^{-1})^{-1}$) using a calibration constant and a range-squared adjustment. The hydrometeor scattering in each velocity bin v can be expressed as [1]

$$S_{obs}(v) = S_{air}(v - \overline{\omega}) * S_{drizzle}(v) + n \tag{1}$$

where $S_{air}(v - \overline{\omega})$ is the air motion turbulent broadening that includes the vertical air motion $\overline{\omega}$, the symbol $*$ represents the convolution function, $S_{drizzle}(v)$ is the drizzle drop reflectivity spectral density, and n is random noise.

The turbulent broadening is assumed to be a Gaussian-shaped probability distribution [2] and can be expressed as

$$S_{air}(v - \overline{\omega}) = \frac{1}{\sqrt{2\pi}\sigma_{air}} exp\left[\frac{-(v-\overline{\omega})^2}{2\sigma_{air}^2}\right] \tag{2}$$

where σ_{air}^2 represents the turbulent broadening variance.

The drizzle spectrum spectral density is modeled as

$$S_{drizzle}(v)\Delta v = N(v)\sigma_b^{Ka}(v)\Delta D(v) \tag{3}$$

where Δv is the velocity bin resolution ($m\ s^{-1}$ and constant for all velocity bins), $N(v)$ is the number concentration at each velocity bin (droplet number per unit volume per droplet diameter), $\sigma_b^{Ka}(v)$ is the backscattering cross-section at Ka-band determined from T-Matrix calculations [3] ($mm^6\ m^{-3}$), and $\Delta D(v)$ is the drizzle diameter breath at each velocity bin v (mm). Note that both sides of (3) have units of reflectivity factor ($mm^6\ m^{-3}$) so that their sum across all velocity bins is the total reflectivity factor that is typically reported by cloud radars.

The retrieval method aims to estimate $N(v)$ without assuming a functional shape (i.e., Gamma or log-normal shape distributions). Combining (1), (2), and (3), the mathematical representation of the retrieved $N(v)$ can be expressed as

$$N(v) = \frac{\Delta v}{\sigma_b^{Ka}(v)\Delta D(v)} S_{air}(v - \overline{\omega}) \otimes S_{obs}(v) \tag{4}$$

where the symbol $\otimes$ represents the deconvolution function.

After estimating $N(v)$ over the uniformly spaced velocity bins v, a fall speed to drizzle droplet diameter relationship is used to map $N(v)$ to $N(D)$.

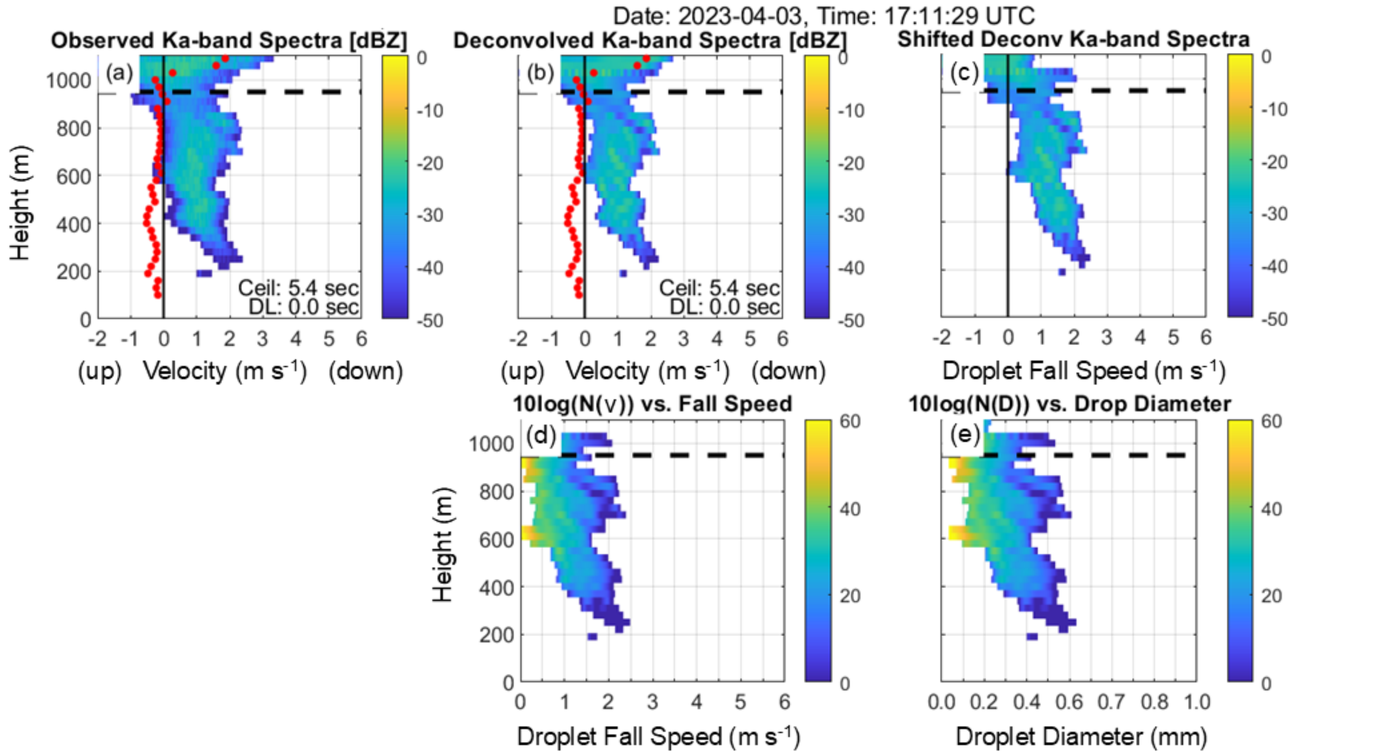

Figure 1. Example of the retrieval method for a profile collected on April 3, 2023, at 17:11:29 UTC. (a) Observed Ka-band radar reflectivity spectral density $S_{obs}(v)$ (color shading) in units of 10log(mm^6 m^{-3} (m s^{-1})$^{-1}$), Doppler lidar air motion estimate (red dots), and ceilometer cloud base estimate (black dashed line). (b) Deconvolved spectra, which is equivalent to $S_{air}(v)\otimes S_{obs}(v)$. (c) Deconvolved spectra shifted by the Doppler lidar air motion estimate, which is equivalent to $S_{air}(v-\overline{\omega})\otimes S_{obs}(v)$. (d) Number concentration in each velocity bin $N(v)$. (e) Number concentration in each droplet diameter $N(D)$. Time differential for the ceilometer and Doppler lidar measurements relative to the Ka-band profile are shown in panels (a) and (b).

I. Results

The retrieval method is best described through an example. Fig. 1a shows an observed Ka-band reflectivity spectral density profile $S_{obs}(v)$ (color shading), Doppler lidar radial velocity (red dots), and ceilometer cloud base (black dashed line) from April 3, 2023, at 17:11:29 UTC. The ceilometer cloud base is near 950 m.

The first step in the retrieval method is deconvoling the turbulent broadening effects from $S_{obs}(v)$. Fig. 1b shows the deconvolved Ka-band spectra where the air motion variance σ^2_{air} at each range gate during this 4-s dwell was estimated using the variance of Ka-band mean velocities over 60-s and the scaling method described in [4].

The second step in the retrieval method shifts the deconvolved spectra by the Doppler lidar air motion estimate and is shown in Fig. 1c. Note that the convolution and shift could be performed in one step, as shown in (4), but the steps were performed sequentially.

The third step in the retrieval method scales the deconvolved and shifted reflectivity spectral density into number concentration at each droplet fall speed and is shown in Fig. 1d.

The last step in the retrieval method converts $N(v)$ to $N(D)$ by changing the horizontal axis from droplet fall speed to drop diameter. $N(D)$ is shown in Fig. 1e.

II. Next Steps

This example showed that, for this specific profile, small drizzle droplets were present just below cloud base. As the drizzle droplets fell, the discrete distribution became narrower and mean droplet size increased. We are performing Lagrangian modeling to quantify the roles of evaporation and sedimentation in this profile and in hours of other drizzling profiles. Also, we are performing error analyses to quantify uncertainties for the retrieved number concentrations.

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