Hybrid Quantum-Classical Algorithms for Satellite Swarm Optimization in Non-Terrestrial Networks

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***Abstract*—This paper presents an efficient hybrid quantum-classical methodology for synthesizing swarm-based antenna arrays in 6G non-terrestrial networks. The approach jointly optimizes satellite element positions and amplitude weights while adhering to engineering constraints like power limitations. The work effectively handles nonlinear objectives and constraints, demonstrating superior sidelobe level (SSL) minimization for large-scale 100-element swarm-based antenna arrays.**

I. Introduction

In the realm of 6G non-terrestrial wireless networks, satellite swarms present a promising solution. The swarm architecture enhances scalability and fault tolerance while substantially reducing production and launch costs [1]. Swarm-based antenna arrays consist of multiple small satellites, each acting as an antenna element, spaced a few wavelengths apart. As shown in Fig. 1, these arrays can use aperiodic patterns like Fermat's spiral geometry or quasi-random arrangements to mitigate grating lobes [2]. Such a design concept aligns with the principles of sparse antenna arrays in the literature. However, optimizing these large-scale systems presents substantial computational challenges due to nonlinear objectives and complex engineering constraints.

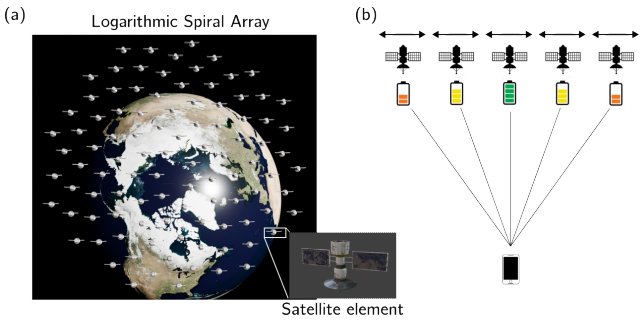


Fig. 1. (a) Swarm-based satellite antenna array in Logarithm Spiral Array (LSA) configuration. (b) Joint optimization of satellite positions and power allocation for effective beamforming towards user equipment.

Existing work primarily focused on optimizing the element positions of swarm antennas with uniform excitations [1], [3]. A recent study explored a quantum-inspired algorithm for optimizing antenna coefficients in sparse antenna arrays, but it assumed a fixed spatial distribution of antenna elements [4]. These approaches, while valuable, do not address the simultaneous optimization of both antenna positions and amplitudes.

In this paper, we introduce a novel hybrid quantum-classical optimization approach aimed at minimizing the maximum sidelobe level (SLL) of swarm antenna arrays. Inspired by the Graver Augmented Multiseed Algorithm (GAMA) [5], our method effectively integrates quantum and classical computing techniques to solve the nonlinear integer program inherent in the optimization of the array's objective function. This approach enables the joint optimization of both antenna positions and amplitude weights under practical engineering constraints, a concept not previously explored in the field. We believe this work opens up new possibilities and provides a valuable contribution to the field of swarm antenna array research.

II. Methodology

We formulate the joint optimization problem for the satellite swarm-based antenna array as a nonlinear integer program. Given a bounded space with 1,000 possible antenna locations, our objective is to determine the optimal configuration for $M = 100$ antennas and their corresponding power allocations. The objective function f minimizes the maximum sidelobe level (SLL) of the radiation pattern by penalizing any far-field radiation (FF) exceeding the target (SLL_t).

$$\min_{\vec{x}} f(\vec{x}) = \begin{cases} FF(\vec{x}) - SLL & \text{if } FF(\vec{x}) > SLL_t \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

f is parameterized by the decision variable vector $\vec{x}$, which consists of two components: $\vec{x}_a$ and $\vec{x}_p$. The vector $\vec{x}_a$ contains the power allocations for the $M = 100$ satellites, while $\vec{x}_p$ is a binary indicator vector corresponding to the $N = 1000$ potential positions, with each element set to 1 if a satellite occupies that position and 0 otherwise.

$$\vec{x} = [\vec{x}_a, \vec{x}_p], \quad x_a \in [0, 7], \quad x_p \in [0, 1] \tag{2}$$

To ensure practical deployment of the swarm-based antenna array, three key engineering constraints are imposed. The power constraint, given by $\sum_a^M x_a = 3M$, limits the total power allocated to the $M = 100$ antennas. The position selection constraint, $\sum_p^N x_p = M$, ensures exactly M positions are selected from the 1,000 possible sites. Additionally, a proximity constraint prevents antennas from being too close to each other by grouping nearby positions into clusters, allowing only one antenna per cluster, $\sum_p^{\text{cluster}} x_p \leq 1$.

All proposed solutions must satisfy these constraints to be considered feasible. We can recast the constraints into a linear system of equations $\mathbf{A}\vec{x} = \vec{b}$, where $\mathbf{A}$ is a 2-dimensional integer matrix. For brevity, the full expression of $\mathbf{A}$ and $\vec{b}$ are omitted. This linear system indicates that all feasible solutions must satisfy the equation, forming a lattice $L(A)$:

$$L(\mathbf{A}) = \{\vec{x} \mid \mathbf{A}\vec{x} = \vec{b},\ \vec{x} \in \mathbb{Z}^t,\ \mathbf{A} \in \mathbb{Z}^{s \times t}\} \tag{3}$$

Lattice $L(\mathbf{A})$ is visualized in Fig. 2, where a grid of points represents all potential solutions and feasible solutions are

represented as black points. In the offline stage, our approach begins by identifying $L(\mathbf{A})$ and the update directions that maintain the feasibility of the initial solution, known as the Graver basis $G(\mathbf{A})$. In the online stage, these Graver bases are used to perform a random walk, guiding the search for the optimal solution.

$$G(\mathbf{A}) = \{\vec{x} \mid \mathbf{A}\vec{x} = \mathbf{0},\ \vec{x} \in \mathbb{Z}^t,\ \mathbf{A} \in \mathbb{Z}^{s\times t}\} \setminus \{\mathbf{0}\} \quad (4)$$

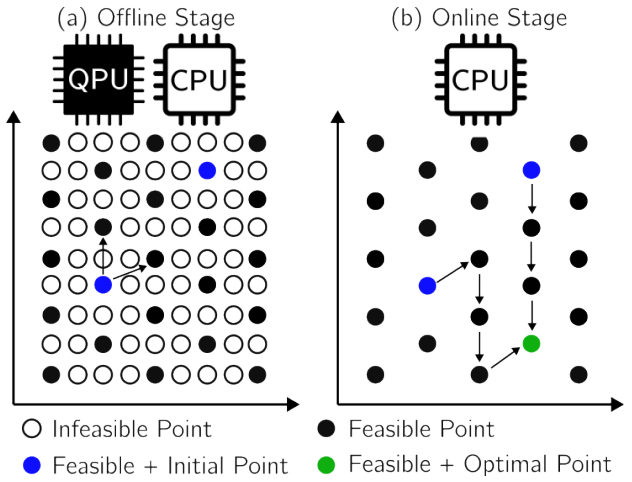


Fig. 2. (a) At the offline stage, initial solution on lattice $L(\mathbf{A})$ and Graver basis $G(\mathbf{A})$ are computed. (b) At the online stage, a random walk method with $G(\mathbf{A})$ finds the optimal solutions.

Nevertheless, finding $G(\mathbf{A})$ and $L(\mathbf{A})$ is NP-hard [6]. In the literature, quantum annealing has been used to address this problem [5], but its applications have mostly been restricted to small-scale problems due to limitations in quantum hardware. In this paper, we propose a hybrid classical-quantum algorithm. First, we use simulated annealing [7] on classical computers to identify good initial solutions for $G(\mathbf{A})$ and $L(\mathbf{A})$. Subsequently, we apply quantum annealing to re-optimize subsets of the problem variables, generating new instances of $G(\mathbf{A})$ and $L(\mathbf{A})$. This approach elegantly leverages quantum computing capabilities to explore the solution space more efficiently.

At the online stage, our primary goal is to use $G(\mathbf{A})$ to guide the initial feasible solution on $L(\mathbf{A})$ toward the optimal solution. We employ a simulated annealing approach, diverging from the greedy or depth-first search methods found in current literature [5]. By utilizing the Metropolis algorithm, our approach enables the initial feasible solutions to navigate toward optimality while effectively overcoming energy barriers between sub-optimal solutions.

III. Numerical Results

In this section, we demonstrate the effectiveness of our proposed method for minimizing the maximum SLL of a swarm of satellites. As a benchmark, we use the Logarithmic Spiral Array (LSA), which is known for providing low SLL in sparse arrays [1]. Using our methodology, we present two sets of solutions: the first optimizes only the selection of antenna positions with uniform amplitude weights, while the second jointly optimizes both the antenna positions and amplitude weights.

As shown in Fig. 3, optimizing the antenna positions alone reduces the maximum SLL from $-17\,\mathrm{dB}$ to $-18.1\,\mathrm{dB}$. Moreover, when we incorporate amplitude optimization, we achieve further reduction in the SLL to $-20\,\mathrm{dB}$. These results underscore the effectiveness of integrating both position and amplitude optimization to achieve superior performance in satellite swarm arrays.

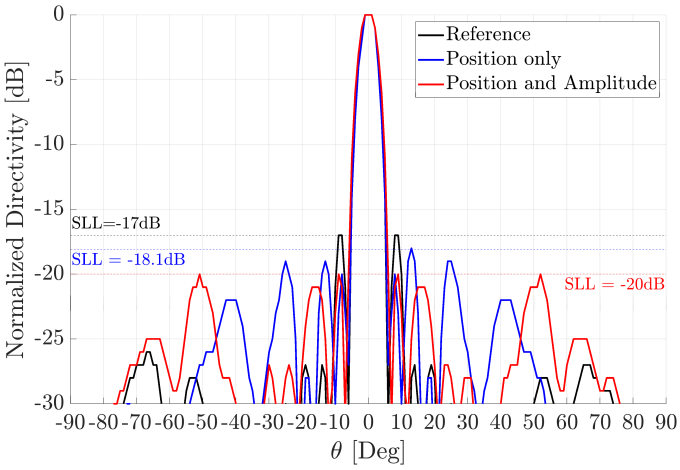


Fig. 3. Comparison of Radiation patterns from LSA and the proposed work.

IV. Conclusion

In this work, we introduce a unique hybrid quantum-classical optimization framework that reframes the synthesis of swarm-based satellite arrays into a non-linear integer programming problem. By leveraging the strengths of both classical and quantum computing, our approach enables the joint optimization of antenna positions and amplitudes under practical engineering constraints. We envision this framework as a solid foundation for future extensions, particularly in scaling up to larger satellite networks and exploring further applications in 6G non-terrestrial communication systems.

References

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