

# Voltage Pulse Propagation on a Dispersive Microstrip Transmission Line

Katherine Aho and Kurt E. Oughstun  
 Department of Electrical & Biomedical Engineering  
 College of Engineering & Mathematical Sciences  
 University of Vermont  
 Burlington, Vermont 05401  
 e-mail kaho@uvm.edu

**Abstract**—The influence of Debye model material dispersion on transmission line dispersion using the Getsinger model is described with regard to voltage pulse propagation along the line as well as the inverse problem of the determination of the medium properties from the propagated pulse behavior.

## I. INTRODUCTION

The propagation of a voltage pulse along a dispersive attenuative transmission line is a problem of fundamental interest in electrical engineering [1] with application to both remote sensing for material identification, such as in time-domain reflectometry (TDR), and environmental vulnerability, such as that found in power lines [2]. The focus of the analysis presented in this paper is on the influence of the dielectric material properties of the substrate on the effective permittivity of a single microstrip transmission line as here described by the so called Getsinger model [3].

## II. TRANSMISSION LINE ANALYSIS

With reference to the geometry and notation of Fig. 2 of Getsinger [3] and following the analysis given there, consider a single microstrip transmission line of width  $W$  on a nonmagnetic dielectric substrate of thickness  $b$  with relative permittivity  $\epsilon_s$ . Above the microstrip line is air (vacuum) with  $\epsilon = 1$ . Microstrip propagation is considered here as a single longitudinal-section electric (LSE) mode approximated as that in a parallel plate transmission line with dielectric permittivity  $\epsilon_s = \epsilon_s(\omega)$ , width  $2s$ , and height  $b$ , connected to other parallel-plate transmission lines with unit relative permittivity, width  $a'$  and height  $b'$  with  $a'/a = b'/b$ .

Within this approximation, the effective relative dielectric permittivity of the microstrip line is found to be given by [3]

$$\epsilon_e(\omega) = \epsilon_s(\omega) - \frac{\epsilon_s(\omega) - \epsilon_{e0}}{1 + G(\omega^2/\omega_p^2)} \quad (1)$$

with  $\epsilon_{e0} = \epsilon_e(0)$  the zero-frequency microstrip relative effective permittivity, where

$$\omega_p = \pi \frac{Z_0}{\mu_0 b} \quad (2)$$

with  $Z_0$  the zero-frequency characteristic impedance of the microstrip,  $\mu_0$  the magnetic permeability of vacuum, and

$$G = \frac{\pi^2 [(\epsilon_{e0} - 1) + (b'/b)^2(\epsilon_s - \epsilon_{e0}) (\epsilon_{e0} - 1)(\epsilon_s - \epsilon_{e0})]}{12 \epsilon_{e0}(\epsilon_s - 1)^2} \quad (3)$$

Notice that  $G = G(\omega)$  depends upon the frequency through the substrate dispersion  $\epsilon_s = \epsilon_s(\omega)$ . However, the Getsinger model [3] does not capture the attenuation of the medium.

The Debye model is implemented in order to consider a substrate with attentive properties. At low frequencies, the material dispersion of the substrate is described by the Debye model [4], [5]

$$\epsilon_s(\omega) = \epsilon_\infty + \frac{\epsilon_{s0} - \epsilon_\infty}{1 - j\omega\tau}, \quad (4)$$

where  $\epsilon_{s0} = \epsilon_s(0)$  is the static relative permittivity of the substrate material,  $\epsilon_\infty \geq 1$  is the high-frequency ( $\omega \gg 1/\tau$ ) limiting value of the relative permittivity, and where  $\tau$  is the effective relaxation time of the material dispersion. Estimates of the model parameters  $\epsilon_{s0}$ ,  $\epsilon_\infty$ , and  $\tau$  from observed pulse distortion should be sufficient to identify the substrate material (e.g. the percentage of water present in soil).

## III. NUMERICAL RESULTS

Figure 1 illustrates the real and imaginary parts of the material dispersion of the substrate as described by the Debye model (4). These results illustrate the extent to which the relaxation time  $\tau$  influences both the real and imaginary parts of the Debye model permittivity.

Of central interest here is the behavior of the effective relative dielectric permittivity of the microstrip line when a Debye model of the substrate dispersion is included. Veghte and Balanis [1] present several propagated pulse results, all using a constant value for the substrate dielectric  $\epsilon_s$ . Of interest in future research is how material dispersion in the substrate will influence these results. Notice that without attenuation, the effective relative dielectric permittivity of the microstrip line increases towards the limit of the value of the substrate.

Figs. 2 and 3 present a comparison of the effective relative dielectric permittivity of the microstrip line with a constant substrate dielectric permittivity with  $\epsilon_s = 10.2$  and with that for the Debye model substrate for several values of the

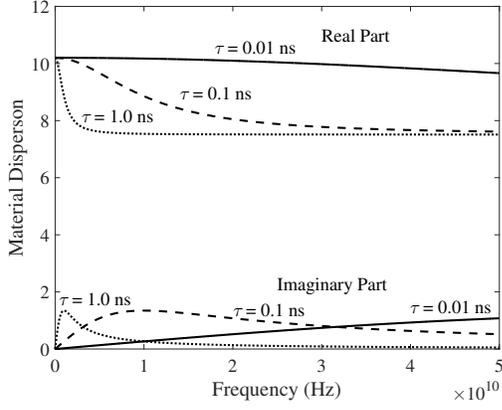


Fig. 1. Real and imaginary parts of the Debye model material dispersion of the substrate with  $\epsilon_{s0} = 10.2$  and  $\epsilon_{\infty} = 7.51$  for several values of the relaxation time  $\tau$ : solid curve ( $\tau = 0.01ns$ ), dashed curve ( $\tau = 0.1ns$ ), dotted curve ( $1.0ns$ ).

relaxation time. The influence of attenuation in the Debye model substrate is prevalent. There is an overall decrease in the magnitude of the real part of the transmission line permittivity as the relaxation time  $\tau$  increases. Fig. 3 also indicates the extent to which the imaginary part influences the attenuation in the transmission line, noting that there is zero attenuation in the Getsinger model for the case of a constant valued substrate permittivity.

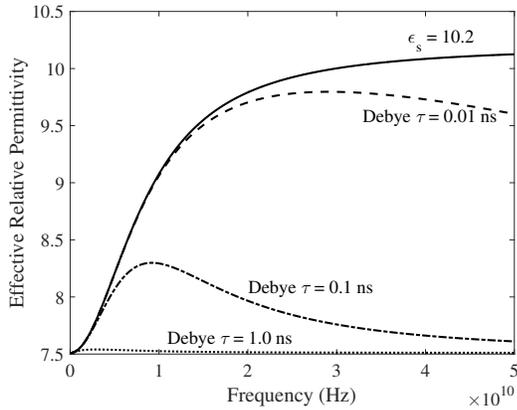


Fig. 2. Comparison of the real part of the effective relative dielectric permittivity of the microstrip line between the constant valued substrate  $\epsilon_s = 10.2$  (upper solid curve) and the Debye model substrate  $\epsilon_s(\omega)$  for several values of the relaxation time  $\tau$ : dashed curve ( $\tau = 0.01ns$ ), dot-dashed curve ( $\tau = 0.1ns$ ), dotted curve ( $1.0ns$ ).

#### IV. FUTURE WORK

Future work will investigate both the effects of substrate dispersion on voltage pulse propagation in a coupled microstrip transmission line as well as the inverse problem on estimating the Debye material properties of the substrate from measurements of the propagated pulse behavior along the transmission line.

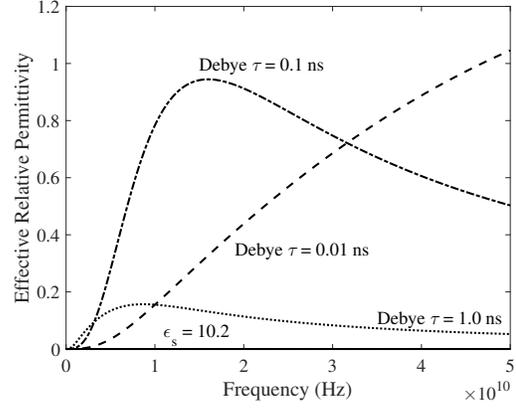


Fig. 3. Comparison of the imaginary part of the effective relative dielectric permittivity of the microstrip line between the constant valued substrate  $\epsilon_s = 10.2$  and Debye model substrate  $\epsilon_s(\omega)$  for several values of the relaxation time  $\tau$ : dashed curve ( $\tau = 0.01ns$ ), dot-dashed curve ( $\tau = 0.1ns$ ), dotted curve ( $1.0ns$ ).

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