Lossy Beam Generation of Circular Arrays

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***Abstract*—This work examines the characteristic modes and measurement of various circularly distributed array topologies in which element radiators are used independently to deliver both sum and difference beams under lossy conditions. An associated moment generating function is derived, such that analytical patterns use even-odd symmetries for sum difference beam behavior. This approach generalizes the Fourier probabilistic methods by using the Laplace transform to analyze statistical averages catering to the degenerating effects of pattern behavior that is influenced by the environment.**

I. Introduction

A collection of individual elements forming a cluster can be connected to form ad-hoc antenna arrays. If each stand-alone transmit/receive module is phased appropriately, a variety of distributions can be applied to create multiple simultaneously-steerable beams with tapered attributes. This pattern behavior can be helpful in many tracking applications, specifically for amplitude monopulse scanning techniques.

II. Problem Formulation

The characteristic function approach (Fourier analysis) [1] – [6] is generalized by finding the moment-generating function (MGF) for this probability distribution. In probability theory, the MGF (when it exists) provides an alternative description to a random system and can be used to compute successive probability moments quickly. Mathematically, it closely resembles the bilateral Laplace transform [7]:

$$M_{G,X}(\Psi) = \int_{-\infty}^{\infty} f_X(x) e^{jx\Psi} dx = L_B\{f_X(x)\}(-\Psi) \quad (1)$$

If $\Psi = \alpha + j\beta$ were taken to be a complex variable, the real part α could be thought of as attenuation in the array factor. The moment generating function evaluated along the imaginary axis ($\alpha = 0$) is then the characteristic function (Fourier transform) inside a lossless medium. Therefore, evaluating the moment generating function along contours in the complex plane can account for differing environmental impacts on the array factor. Physically, α should be taken to be negative to represent attenuation accurately.

For the CCF, solving (1) for the generalized n-th modes yields the appropriate mode solutions. This result was done in two cases to represent even and odd behavior in the excitation:

$$M_E(\Psi) = \int_{-1}^{1} f_X(x) e^{jx\Psi} dx = \Gamma(1+n/2) I_{n/2}(\Psi)/\Psi^{n/2} \quad (2)$$

$$M_O(\Psi) = \int_{-1}^{1} f_X(x;n) e^{jx\Psi} \operatorname{sgn}(x) dx = \Gamma\left(1+\frac{n}{2}\right)\frac{L_{n/2}(\Psi)}{\Psi^{n/2}} \quad (3)$$

Table 1. Enumerated Even (Sinhc) and Odd (CoSinhc) Modes for n = -1 to 3. The functions in $i_n(\Psi)$ and $l_n(\Psi)$ are the modified spherical Bessel and Struve functions [8 – 9].

Modes	n=-1	n=0	n=1	n=2	n=3	Shell
$\Lambda(\Psi)$ Even	$\cosh(\Psi)$	$I_0(\Psi)$	$i_0(\Psi)$	$2I_1(\Psi)/\Psi$	$3i_1(\Psi)/\Psi$	$3(i_0(\Psi)-i_1(\Psi)/\Psi)/2$
$\Lambda(\Psi)$ Odd	$\sinh(\Psi)$	$L_0(\Psi)$	$l_0(\Psi)$	$2L_1(\Psi)/\Psi$	$3l_1(\Psi)/\Psi$	$3(l_0(\Psi)-l_1(\Psi)/\Psi)/2$

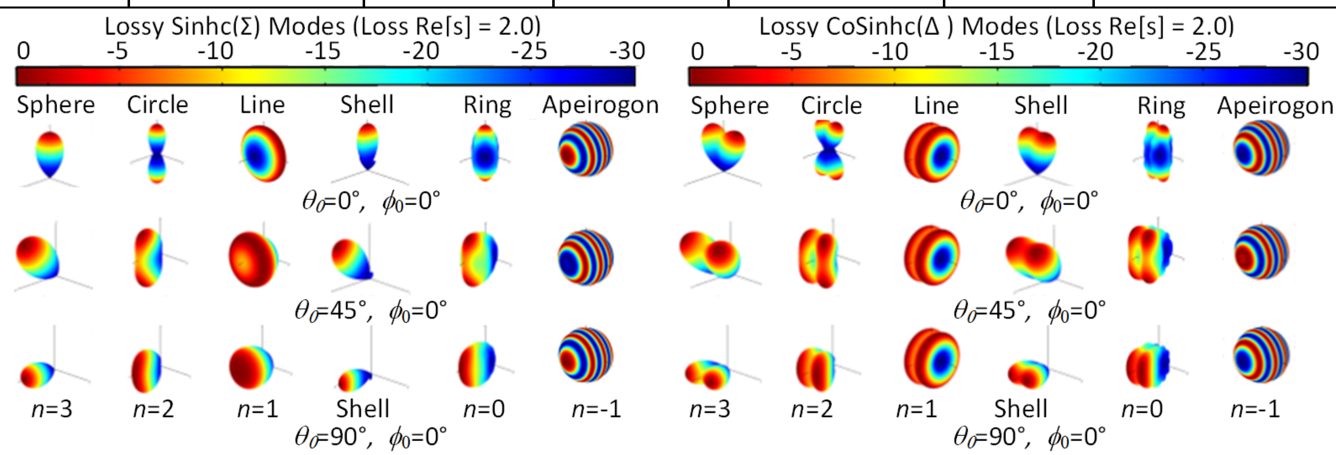

Fig. 1. Array factor (Σ and Δ) pattern comparisons scanned from the zenith to the meridian.

The special functions $I_n(\Psi)$ and $L_n(\Psi)$ are the modified Bessel and Struve functions [8], respectively. Since these functions are closely related to the hyperbolic sines and cosines (mathematical generalizations), the n-th mode solutions in Equations (2) and (3) form a family of Sinhc (Even) and CoSinhc (Odd) characteristic modes for the circular canonical family (CCF) distribution.

III. Numerical Analysis

The derived modal solutions enumerated in Table 1 are the generalized lossy characteristic modes of the CCF. Since Ψ is taken to be a complex argument, the effective normalized radiation patterns accounting for losses can be found by evaluating the modes along specific contours in the complex Ψ plane. This analysis is also similar in the approach to analyzing filter transfer functions in the Laplace s domain. The modal functions are analogous to filter transfer functions and contain poles and zeros; the addition of environmental loss shifts the poles and zeros off the imaginary axis, directly modifying the array's observed frequency response. The Sinhc_n and CoSinhc_n modes are evaluated along specific contours, which include the loss parameter α and determine the frequency response within a particular environment.

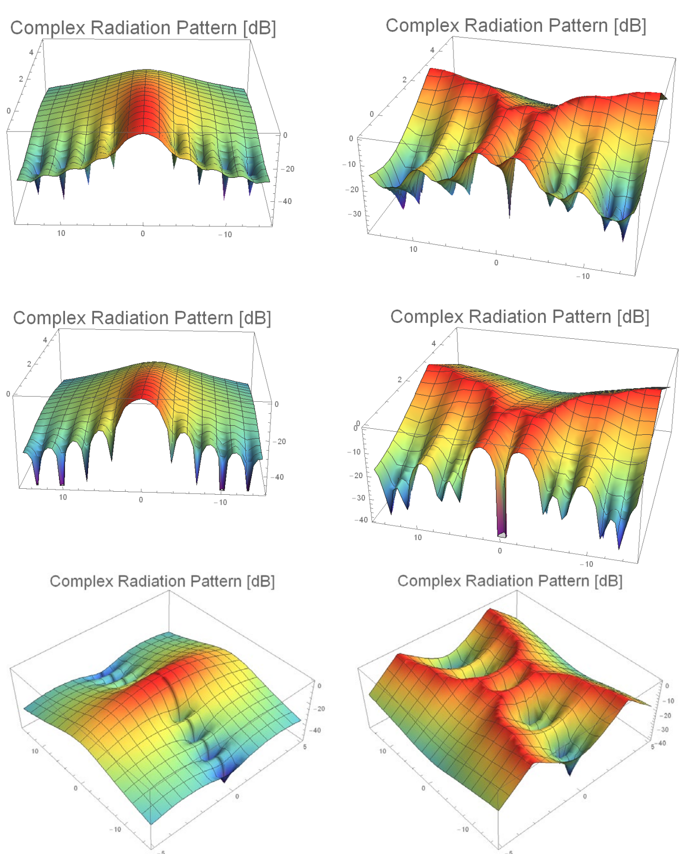

Fig. 2 Complex Radiation Pattern from Moment Generating Function.

Fig. 2 – 3 shows the normalized mean radiation pattern of the CCF modes. The lossless case is the same as the characteristic function (Fourier transform) method, showing the even and odd radiation modes. The loss was added into the pattern by evaluating the moment-generating functions along three specific contours; in each case, the attenuation α is assumed to be peaked along the j_0 (or real) line. To accurately model real environments, α is tapered down so that the net effect is a tapering of the radiation patterns. The lossy cases in Fig. 2 – 3 show an apparent attenuation of the mean pattern. This result is akin to shifting the poles and zeros away from the imaginary axis in analog filter design. As the poles are shifted, the main lobe is suppressed while the sidelobes become envelopes with their nulls disappearing. Moreover, the odd mode patterns quickly lose their difference-pattern behavior as the loss parameter increases. The net effect for increasing α is that the lobed patterns flatten, and the nulls peak up.

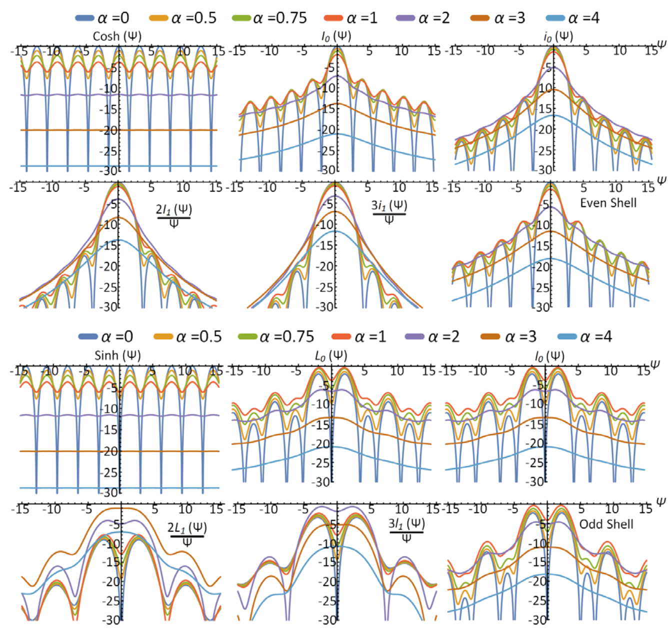

Fig. 3. The normalized mean pattern of the even (left) and odd (right) CCF modes in a lossless environment in Ψ space for various values of α.

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