Remarks on the Parabolic Equation Model for Waves in Random Media

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One of the most important contributions of V.I. Tatarskii is the introduction and development of the parabolic equation model (PEM) for waves in random media (WRM). Most of the theories for WRM are based on perturbation methods that place stringent constraints on the magnitude of refractive index fluctuations. The PEM was hence introduced as a strong fluctuation theory. An important merit of the theory is that one can obtain closed equations for moments of wave functions of any order. This model is based on the following three assumptions: (a) wave propagation is predominant in one direction (negligible backscatter), (b) the refractive index fluctuations obey Gaussian statistics, and (c) the random medium is delta correlated along the direction of propagation. In spite of the restrictions imposed by these assumptions, PEM has been quite successful in numerous applications since 1970.

The validity and rationale behind the assumptions in PEM were convincingly explained by Tatarskii. Accordingly, the wavelength should be much smaller than the correlation length of the medium. The upper bound of the propagation distance is determined by the mean free path. This in turn places restrictions on the magnitude of fluctuations (although not as severe as in perturbation theories). However, it is still not clear how to unambiguously quantify the regime of applicability of the PEM.

One fruitful way to better understand the PEM is to identify the important parameters of the problem with reference to a common scale, introduce scale separation, and adopt an asymptotic analysis in the region of interest. By following this procedure one can show how the original Helmholtz equation reduces to a parabolic equation, and further becomes equivalent to an Ito-Schrodinger equation (ISE) in the limit of large propagation distance. This is a distributional equivalence, and the convergence is in terms of statistical distributions. From the ISE one can proceed to obtain closed equations for wave function moments of any order. Note that there is no need for explicitly imposing assumptions of forward propagation, delta correlation, or even Gaussian statistics in this approach. Hence we get a different perspective and a more unified understanding of the PEM and its domain of validity. Although WRM has been widely studied in numerous applications, the theoretical issues that one encounters are highly nontrivial. For instance, there still remain several open questions on the PEM, the wave function statistics, and the different regimes involved in them.