Evaluation of 6-D Reaction Integrals via Double Application of the Divergence Theorem

Javier Rivero[(1)], D.R. Wilton* [(2)], F. Vipiana[(1)], and W.A. Johnson[(3)]
(1) Politecnico di Torino, Torino, 10129, Italy
(2) University of Houston, Houston, 77204, TX, USA
(3) Electromagnetics Consultant, Jemez Springs, 87025, NM, USA

The efficient and accurate evaluation of singular or near-singular double volume reaction integrals is crucial to the method of moments (MoM) solution of the volumetric integral equations (VIEs) occurring in a large class of electromagnetic (EM) problems ranging from modeling propagation and integrated circuits to clinical medical devices. The usual evaluation strategy for these double volume (6-D) integrals is to first integrate singular potential integrals over the source volume, and then to apply standard cubature rules for the remaining test integral, which often does not account for singularities introduced from the first integral. Here, we propose instead applying the divergence theorem to both the source and test integral, resulting in an inner, double radial integral that may be evaluated in closed form for homogeneous scalar Green's functions and linear basis and test functions, plus double surface (4-D) integrals over source and test element boundary face pairs. Other similar dimensional-reduction approaches appear in recent literature, but often not exploiting the closed form nature of the radial integrals. Though different, our approach appears most similar to that of (Bleszynski et al., IEEE T-AP, 61, 3642–3647, 2013). In the latter, one first expresses the integrand as a Laplacian (requiring double radial integration as well), which is then converted to double surface integrals. Though the two formulations appear quite different, numerical results show that, without further acceleration, their convergence rates with respect to quadrature sampling density are very similar. Reaction integrals between coplanar faces always vanish in our formulation, however, which is a particularly advantageous feature in evaluating self-element pair reactions.

The slower observed convergence of surface reaction integrals for element pairs with one or more vertices in common seems likely caused by the presence of low-order integrand singularities in face-pair reaction integrands. Good integral transformations for accelerating the evaluation of such integrals have yet to be developed, but we find that a simple singularity subtraction scheme based on a hybrid analytical-numerical evaluation of the corresponding static integrand form significantly accelerates their convergence. Presented numerical results demonstrate these improved convergence rates.