

# Extraction of Nonlinear X-Parameters from FDTD Simulation of a One-Port Device

Joshua M. Kast and Atef Z. Elsherbeni  
Electrical Engineering Department  
Colorado School of Mines, Golden, CO, USA  
jkast@mines.edu and aelsherb@mines.edu

**Abstract**—A formulation is developed for X-Parameters extraction of nonlinear devices when a finite-difference time-domain electromagnetic simulation is used. A simulation domain containing a diode is tested, and several excitation waveforms are applied to capture the diode’s nonlinear behavior. Post-processing of simulation results allows for the extraction of X-Parameter values, and a good agreement is obtained when comparing the results from the developed procedure and a commercial simulator.

## I. INTRODUCTION

The finite-difference time-domain (FDTD) technique is a popular approach for simulating the behavior of RF and microwave devices [1]. In this work, we present a formulation to extend the traditional FDTD technique by extracting X-Parameter [2] information from the time-domain simulation results. Applications of this technique include the verification of novel FDTD formulations for nonlinear devices, as well as the simulation of complex structures which comprise both nonlinear components and 3D geometries.

Based on the polyharmonic-distortion method, the X-Parameter formulation is a behavioral model for nonlinear RF and microwave devices. The X-Parameter model is a superset of the well-known S-parameters for passive microwave devices [2]. As with S-parameters, X-Parameters can serve as a common format for the interchange of data from microwave device measurements and simulations.

By extracting X-Parameter data from the results of FDTD simulations, we enable the simulation results to be compared with other sources of X-Parameter data, including laboratory measurements, manufacturer specifications, or other simulation results. In addition, once the X-Parameter model has been extracted from the FDTD results, this model may be incorporated into network-level simulations of a microwave system. This is analogous to the incorporation of measured or simulated S-parameter results into a simulation of an RF network.

## II. METHODS

To extract X-Parameters from the results of FDTD simulations, we must first design a set of waveforms which will excite a device’s fundamental frequency of operation as well as several harmonic frequencies. An FDTD domain is then developed and simulated using individual waveforms from the previously developed set. Finally, time-domain voltage and current results from the FDTD simulations are post-processed to

calculate X-Parameter results. In this work, we present the use of this technique to simulate the nonlinear operation of a junction diode.

### A. Design of Excitation Waveforms

In their formulation of the X-Parameters, Root et al. [2] define a “large-signal operating point” (LSOP), which is the amplitude, frequency, and phase of a signal used to excite the device under test (DUT) during normal operating conditions. This LSOP is typically selected to match the characteristics of a real-world signal (such as a communications signal), which would be applied to the DUT.

We achieve this LSOP in our simulated DUT by exciting the domain with a time-domain voltage of  $V_{\text{source, LSOP}}(t)$ , where:

$$V_{\text{source, LSOP}}(t) = \Re\{K_0 e^{2\pi j f_0 t}\}. \quad (1)$$

In (1),  $f_0$  represents the fundamental operating frequency of the device, and  $K_0$  is a coefficient used to adjust the amplitude and phase of this stimulus signal. In this work, an  $f_0 = 8.295$  GHz was used, along with  $K_0 = 1$ . In addition to this fundamental frequency, four harmonics were studied ( $N = 4$ ).

To calculate the large-signal  $X_{p,k}^{(FB)}$  parameters for a device, it is only necessary to use a single excitation waveform – a sinusoid with frequency  $f_0$ . For determination of small-signal terms such as  $X_{p,k;q,l}^{(S)}$ , additional excitations are required which contain the frequency  $f_0$  as well as its harmonics.

### B. FDTD Simulation

An FDTD simulation was set up using the formulation described in [1] and implemented in [3]. The circuit shown in Fig. 1 contains a junction diode in series with a  $50\Omega$  resistor and a voltage source. The FDTD implementation of this circuit is shown in Fig. 2: the voltage source occupies the left portion of the figure, and is attached to the diode and resistor by thin plates of simulated perfect electric conductor (PEC) material, shown in red. The PEC plates are 0.1 mm wide, 0.2 mm long, and are separated by 0.1 mm. A Yee cell size of 0.05 mm was used for this simulation. Outside of the domain shown in Fig. 2, an additional 8 cells of air-buffer and 10 cells of perfectly matched layer (PML) boundary condition are present in each direction.

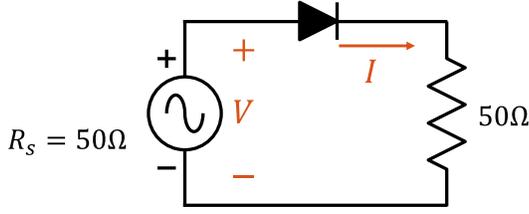


Fig. 1: Schematic of circuit to be simulated: junction diode in series with a 50Ω resistor, supplied by a voltage source with internal resistance of 50Ω.

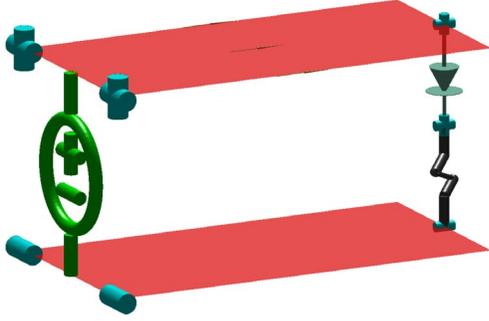


Fig. 2: Simulation domain for circuit in Fig. 1. Parallel conductors are 0.1 mm wide and have 0.1 mm spacing.

### C. Post Processing

In this work, we demonstrate the use of FDTD simulation and post-processing to determine the 1-Port large-signal X-Parameter quantity  $X_{p,k}^{(FB)}$  from experimental results, where  $p$  and  $k$  represent the measurement port and harmonic, respectively.

The first step in the post processing is to convert the voltage and current quantities from the simulation to wave quantities. At each frequency of interest, the time-domain voltage and current values are converted to frequency using a discrete Fourier transform. To determine the phasor voltage quantity for the  $k^{\text{th}}$  harmonic of  $f_0$ , on a time-domain voltage signal  $V$  containing  $N$  measurements taken at port  $p$  with time-step of  $dt$ , we may take:

$$\tilde{V}_{p,k} = \sum_{n=0}^{N-1} V_p(n \cdot dt) e^{2\pi j k f_0 t}, \quad (2)$$

and similarly for current:

$$\tilde{I}_{p,k} = \sum_{n=0}^{N-1} I_p(n \cdot dt) e^{2\pi j k f_0 t}. \quad (3)$$

These phasor quantities are then used to calculate wave parameters, which describe the power entering and leaving the simulated DUT at each frequency. The  $B$  wave, which represents the power travelling out of the device, is found as:

$$B_{p,k} = \frac{\tilde{V}_{p,k} - Z_0 \tilde{I}_{p,k}}{\sqrt{Z_0/2}}. \quad (4)$$

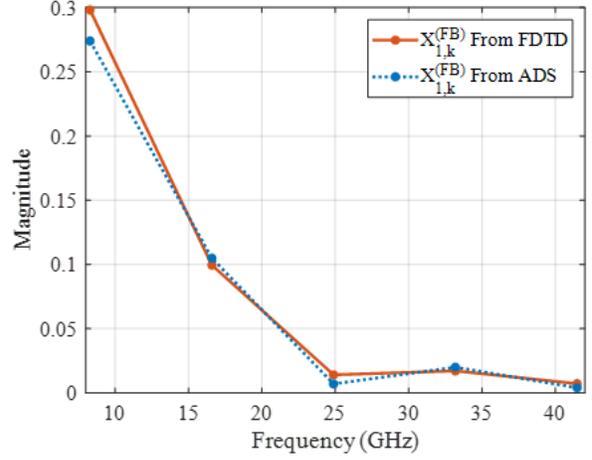


Fig. 3: Calculated values for large signal ( $X^{(FB)}$ ) quantities, with comparison to ADS results.

Once the  $B$  waves are calculated at each port, it is possible to determine the large-signal X-Parameter quantities. These describe the outward-traveling ( $B$ ) waves which are present at each port in response to the LSOP stimulus alone. We calculate  $X_{p,k}^{(FB)}$  by:

$$X_{p,k}^{(FB)} = B_{p,k}, \quad (5)$$

Values of  $X_{p,k}^{(FB)}$  obtained by post-processing FDTD simulation results are plotted in Fig. 3, along with results from simulation of the schematic in Fig. 1 using the ADS simulator from Keysight [4]. Good agreement is seen between the two quantities.

### III. CONCLUSIONS

In this work, we present the successful extraction of large-signal  $X_{p,k}^{(FB)}$  X-Parameter terms from the FDTD simulation of a junction diode. This effort provides the foundation for extraction of other nonlinear parameters from FDTD simulations, and for the FDTD simulation of devices which are defined by their X-Parameters.

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### REFERENCES

- [1] A. Z. Elsherbeni and V. Demir, *The finite-difference time-domain method for electromagnetics with Matlab simulations*. Raleigh: The Institution of Engineering and Technology, 2016.
- [2] D. Root, J. Verspecht, J. Horn, and M. Marcu, *X-Parameters: Characterization, Modeling, and Design of Nonlinear RF and Microwave Components*. Cambridge University Press, September 2013.
- [3] V. Demir and A. Elsherbeni, "Computational Electromagnetics Simulator (CEMS)," software package version 4, veysdemir@gmail.com, August 2020.
- [4] Keysight, Santa Rosa, CA, USA. 2019. Advanced Design System 2019 [Online]. Available: <http://www.keysight.com/en/pc-1297113/advanced-design-system-ads?&cc=US&lc=eng>