Full-Wave Transmission Line Theory (FWTLT) for a Thin-Wire Transmission Line inside a Rectangular Resonator

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***Abstract*—This article presents a solution for coupling an electromagnetic field to a thin wire transmission line in a rectangular resonator, with the wire being loaded at the ends and excited by lumped sources. The mathematical model is based on the Telegrapher's equations as a system of ordinary differential equations. The corresponding parameter matrix for the equation system has complex values, is length-dependent and contains diagonal elements. In contrast to free space, the non-diagonal elements for the transmission line in a lossless resonator are purely real and the diagonal elements are purely imaginary. The applicability of a simple perturbation theory for the parameters is investigated.**

I. Introduction

The coupling of high-frequency electromagnetic field to different kinds of transmission lines is one of the main problems in electromagnetic compatibility. Often the transmission lines are arranged in resonator-like objects, such as computer cases, aircraft fuselages, cars, etc. A change in the geometric environment can radically affect the coupling. The most used numerical methods like MoM, TLM, etc. to solve the problem consider only specific cases and do not provide a general physical understanding. On the other hand, if the wavelength at low frequencies is much greater than the transverse dimension of a transmission line in free space in the so-called classical transmission line approximation (CTLA) is applicable. This approximation provides an analytical solution for the coupling problem and allows a qualitative analysis of the solutions [1].

The CTLA has been generalized at high frequencies by the so-called full-wave transmission line theory (FWTLT), which can be derived from the exact system of mixed-potential integral equations (MPIE) that, in turn, can be derived from the Maxwell equations. The scattered current and the potential along the line are represented as a system of first order differential equations, which is similar to the telegrapher equation system of CTLA [2]–[6]. However, the corresponding parameter matrix **P** has complex values, is length-dependent and contains diagonal elements. The non-classical parts of the parameter matrix, here the diagonal elements and the imaginary parts of the non-diagonal elements, define the radiation of the system. These parameters can be obtained, e.g., from the solutions of the current and the potentials that are excited by the left and right lumped voltage sources (admittance functions).

These response admittance functions can in turned be obtained numerically, e.g. by NEC [5] or analytically using the so-called method of modal parameters [7], [8]. Another analytical method to obtain the parameters matrix in FWTLT is the perturbation theory [7]-[9], [12], [13]. In the modal parameter method, the MPIE equations for a wire of finite-length can be reduced to the infinite system of linear equations by applying the Fourier transformation. The corresponding infinite matrix for the modal inductance per-unit-length and the modal capacitance per-unit length can be obtained as a result of the Fourier transformation for the kernels of MPIE equations. These matrices as well as the connected matrix of the modal impedance per-unit length have a deep physical meaning [7], [8], [10]. In the symmetrical case of a vertical semicircular loop above a perfectly conducting ground, these matrices are diagonal, and one can easily obtain the admittance functions and the matrix of the FWTLT parameters [7]-[8].

II. Description of results

In the present paper, we derive an exact MPIE equation for the current and the potential along a thin wire located in an arbitrary geometric environment with known Green function, where the wire is excited by an EM field. In the case of a transmission line with a symmetrical geometry within a rectangular resonator (see Fig.1), the MPIE can be explicitly

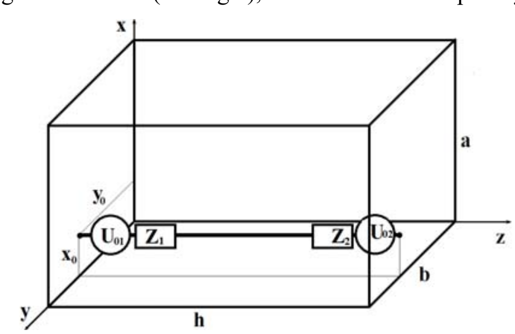
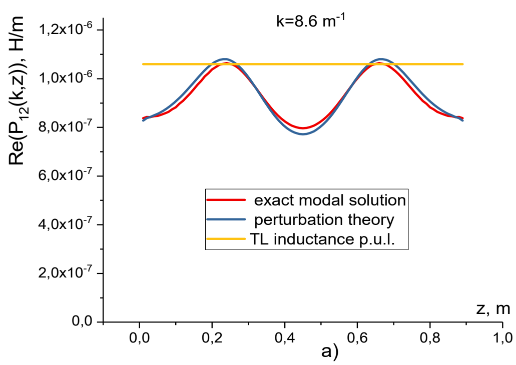


Fig. 1. Loaded thin-wire line with symmetrical geometry in a rectangular resonator (Resonator: a=1.5m, b=1.2 m, h=0.9 m; Transmission line: x_0=9cm, y_0=37 cm, r_0=1 mm).

solved analytically by using the method of modal parameters, including the case of a lumped excitation with a loaded line [14 -15].

Then we generally show that the exact MPIE equation for the current and potential along a thin-wire line excited by an arbitrary terminal lumped source(s) can be reduced to the FWTLT system of first-order differential equations. This system of equations is similar in form to the Telegrapher's equations. As in the case of free space, the parameter matrix of this system $\mathbf{P}$ can be built on the basis of the partial solutions of MPIE with lumped excitations and does not depend on their choice and amplitudes. The matrix $\mathbf{P}(j\omega,l)$ is length dependent and contains diagonal elements. Investigations have shown that, in contrast to a line in free space, in a lossless resonator, the non-diagonal elements are purely real and the diagonal elements are purely imaginary. As in the case of the TL in free space, the parameters show strong changes and small oscillations near terminals (see Fig.2). It can be shown that this is caused by the influence of leaky modes of current [10]. The characteristic length of this spatial dependence is defined by the geometry of the problem. At high frequencies, the parameters along the line oscillate approximately with the period of the wavelength.



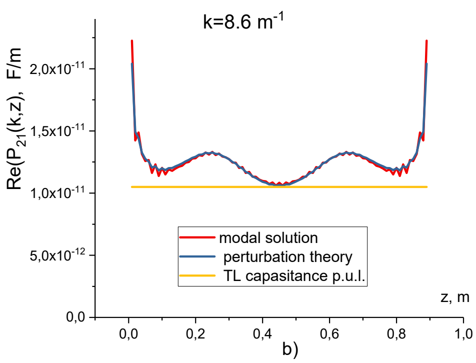


Fig. 2. Length dependence of the P_{12} (inductance-like) and P_{21}(capasitance-like) elements of the parameter matrix for the high-frequency case far from the cavity resonances. $k=8.6m^{-1}$.

Then we have introduced the perturbation theory for the parameters, as in [7], [8], [11], investigated its application using the numerical examples, and analyzed the results. If the frequency is far from cavity resonances, the perturbation theory of first order provides a good description of the parameters. However, near the cavity resonances, the results are not accurate. This is caused by the fact that the zero iteration does not consider the cavity resonances. In future research we would like to improve the perturbation theory for FWTLT parameters. There are two possibilities: 1) consider more complicated zero iterations for the current including at least one resonator mode[16]; 2) continuation of at least two iterations of the perturbation theory in the calculation.

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REFERENCES

[1] F.M. Tesche, M.V. Ianoz, T. Karlsson, EMC Analytical Methods and Computational Models, Willey&Son,1997.

[2] H. Haase and J. Nitsch, Full-wave transmission line theory (FWTLT) for the analysis of three-dimensional wirelike structures, in 14th Int. Zurich Symp. on EMC, pp. 235–240, Feb. 2001.

[3] H. Haase, J. Nitsch, T. Steimetz, Transmission-line super theory: A new approach to an effective calculation of electromagnetic interactions, Radio Science BulletinNr. 307 , Dec. 2003, pp. 33-60.

[4] H. Haase, J. Nitsch, T. Steimetz, "New propagation models for electromagnetic waves along uniform and nonuniform cables", IEEE Trans. on EMC, vol. 46 , Nr. 3 , Aug. 2004, pp. 345 - 352.

[5] K.K. Mei, Theory of Maxwellian circuits, Radio Science Bulletin306 , Sept. 2003, pp. 6-13.

[6] L. Li, Y.-W. Liu, K. K. Mei, and K.-W. Leung, Applications of the Maxwellian Circuits to Linear Wire Antennas and Scatterers, IEEE Trans. Ant. Prop. vol. 54, Nr. 10, Oct. 2006, pp. 2725-2730.

[7] J.B.Nitsch, S.V.Tkachenko, Global and Modal Parameters in the Generalized Transmission Line Theory and Their Physical Meaning, Radio Science Bulletin, **312**, March 2005, pp.21-31.

[8] J.B.Nitsch, S.V.Tkachenko, Propagation of Current Waves along Quasi-Periodical Thin-Wire Structures: Taking Radiation Losses into Account, Radio Science Bulletin, No **322**, September 2007, pp. 19-40.

[9] J.Nitsch, F. Gronwald, G. Wollenberg, Radiating Nonuniform Transmission-Line Systems and the Partial Element Equivalent Circuit Method, Willey, NY, 2009.

[10] J. Nitsch, S. Tkachenko, Physical Interpretation of the Parameters in the Full-Wave Transmission Line Theory, VXV Int. Symp. on Theor. Eng., Lübeck, Germany, 22-24 June 2009. ISBN:978-3-8007-3166-4

[11] J. Nitsch, S. Tkachenko, High-frequency Multiconductor Transmission line Theory, Found. Phys. (2010) 40: 1231-1252.

[12] F.Ossevorth, R.T.Jacobs,and H.-G. Krauthäuser, A full wave description for thin wire structures with TLST and perturbation theory, Adv. Radio Sci.,16, 123-133, 2018, https://doi.org/10.5194/ars-16-123-2018.

[13] R. Rambousky, J.B. Nitsch, H. Garbe, Application of the Transmission-Line Super Theory to Multiwire TEM-Waveguide Structures, IEEE Trans. on EMC, vol. 55 , Nr. 6 , Dec. 2013 , p.1311-1319.

[14] S.V.Tkachenko, R.Rambousky, J.Nitsch, Electromagnetic Field Coupling to a Thin Wire Located Symmetrically Inside a Rectangular Enclosure, *IEEE Trans. on EMC*, vol. 55, Nr.2, Apr. 2013, pp. 334-341.

[15] S.Tkachenko, J.Nitsch, R.Rambousky, Electromagnetic Field Coupling to Transmission Lines Inside Rectangular Resonators, Interaction Notes, Note 623, 2011. http://ece-research.unm.edu/summa/notes/In/IN623.pdf.

[16] S. V. Tkachenko, J. Nitsch, R. Vick, “HF Coupling to a Transmission Line inside a Rectangular Cavity”, in Trans. of URSI Int. Symp. on Electromagnetic Theory, Berlin, Germany, August 16-19, 2010 (ISBN: 978-1-4244-5154-8), pp. 41-44.