Phase-Space Dynamic of Coherent Wave–Particle Interaction in the Radiation Belts

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***Abstract*— Modeling the interaction between coherent whistler mode waves and radiation belt electrons is an important component of space weather dynamics. Two main aspects of the wave-particle interaction are, the amplification of coherent VLF waves by an unstable radiation belt electron distribution and the precipitation and/or acceleration of these particles by the waves. The solution of the full problem requires a numerical self-consistent code which captures both effects simultaneously. Unfortunately, self-consistent codes of nonlinear phenomena are computationally intensive and the results can be challenging to interpret. To quantify the effect of waves on particles, we employ a novel approach wherein the particle trajectories are traced backward in time. The validity of this method is based on conservation of phase space density formalized in Liouville's theorem. The model resolves in high resolution the formation of a depletion in the region of phase-space known as a phase space hole that is associated with nonlinear wave growth.**

I. Introduction

Plasma waves in the magnetosphere play an important role in the dynamic of the Earth's radiation belts. The interaction between these waves and geomagnetically-trapped energetic electrons is a key process in this region. Two main aspects of the wave-particle interaction are, the amplification of the wave by an unstable radiation belt electron distribution [1] and the precipitation and/or acceleration of these particles by the waves [2]. The most fundamental physical description of the interaction requires correctly modeling wave amplification while self-consistently capturing the evolution of the particle distribution in phase space ($\vec{r}, \vec{v}$). The full treatment of amplification and scattering/acceleration is a difficult problem that requires solving Vlasov-Maxwell system of equations. Several authors have used Vlasov-Maxwell solvers to model wave growth [3]; however, measuring particle distribution in the same numerical model can be difficult to do accurately. To quantify time evolution of the particle distribution, many authors considered simplified "no feedback" models where the waves are assumed to be generated by transmitters or lightning [2]. A common method of evaluating the particle distribution is through quasi-linear theory and the calculation of diffusion coefficients [4]. However, the primary disadvantage is that it is formally valid only for small amplitude, incoherent signals. Therefore, the general dynamic of particle phase space distribution interacting with large-amplitude coherent waves may not be correctly handled by quasi-linear theory [5]. Here, we use an efficient characteristic-based solution to the Vlasov equation (Vlasov-Liouville Model) to evaluate the dynamic effect of phase-trapped particles on the phase space distribution function.. Since a large body of previous work has focused on quasi-linear theory, this study is important for evaluating the wave-particle interaction in the regime where strongly nonlinear effects cannot be neglected.

II. Theoretical Background

For simplicity, we consider only parallel propagating (z direction) whistler mode signals. The equations of motion (Lorentz force) used are shown in (1)-(4), which describe the interaction between relativistic (Lorentz factor γ) electrons (with charge q and mass m) and whistler mode waves (E_w, B_w) immersed in a background inhomogeneous magnetic field ($\frac{\partial \omega_c}{\partial z}$).

$$\frac{dz}{dt} = v_\parallel \tag{1}$$

$$\frac{dp_\parallel}{dt} = \frac{q}{m\gamma} B_w p_\perp \sin\varphi - \frac{{p_\perp}^2}{2m\gamma\omega_c}\frac{\partial \omega_c}{\partial z} \tag{2}$$

$$\frac{dp_\perp}{dt} = -q\sin\varphi\,(E_w + v_\parallel B_w) + \frac{p_\parallel p_\perp}{2m\gamma\omega_c}\frac{\partial \omega_c}{\partial z} \tag{3}$$

$$\frac{d\varphi}{dt} = -k(v_{res} - v_\parallel) - \frac{q\cos\varphi}{p_\perp}(E_w + v_\parallel B_w) \tag{4}$$

Where $p_\parallel$ ($v_\parallel$) and $p_\perp$ are parallel and perpendicular components of electron momentum (velocity) relative to the background geomagnetic field. The angle between $p_\perp$ and $-B_w$ is referred to as the gyrophase (φ). When the Doppler-shifted wave frequency ($\omega + kv_\parallel$) experienced by the particle equals a multiple of the gryrofrequnecy (ω_c), the particle can resonant with the wave (wave number k), and the resonance velocity is given by $\frac{\omega_c - \omega}{k}$. An important consequence of equations (1)-(4) is phase trapping and the formation of a separatrix in phase space. If only particles that are close to resonance are examined and the small contribution of centripetal acceleration due to the wave is neglected, equations (2) and (4) can be simplified to (5) and (6).

$$\frac{d\varphi}{dt} = k(v_\parallel - v_{res}) \tag{5}$$

$$\frac{d^2\varphi}{dt^2} - \omega_{tr}^2(\sin\varphi - S) = 0 \tag{6}$$

where ω_{tr} is the trapping frequency and is given by $\sqrt{\frac{q}{m} B_W k v_\perp}$. S is the "collective inhomogeneity factor" and is given by (7) [3].

$$S = \frac{1}{\omega_{tr}^2}\left[\frac{k v_\perp^2}{2\omega_c} + \frac{3v_{res}}{2}\right]\frac{\partial \omega_c}{\partial z} \quad (7)$$

To illustrate the formation of a separatrix, it is useful to examine particle trajectories in the $(v_\parallel, \varphi)$ coordinates. The separatrix divides the trajectories into two regions: trapped and untrapped. Trapped particle trajectories (interior) form closed curves while the untrapped particle trajectories swing around the separatrix (See Fig. 1a).

III. Model Description

The Vlasov-Liouville (VL) numerical model essentially solves the Vlasov equation for a given wave at a particular location along the field. The Vlasov equation governs the evolution of a collisionless plasma in phase space $(\vec{r}, \vec{p})$, The Vlasov equation is shown in (8); the terms in parenthesis corresponds to equations (1)-(4).

$$\frac{\partial f}{\partial t} + \left(\frac{dz}{dt}\right)\frac{\partial f}{\partial z} + \left(\frac{dp_\parallel}{dt}\right)\frac{\partial f}{\partial p_\parallel} + \left(\frac{dp_\perp}{dt}\right)\frac{\partial f}{\partial p_\perp} + \left(\frac{d\varphi}{dt}\right)\frac{\partial f}{\partial \varphi} = 0 \quad (8)$$

Since the Vlasov equation is an advective-type partial differential equation (PDE), information propagates around phase space in a complicated manner. An accurate method of computing the distribution is by using the method of characteristics, which in the context of Vlasov equation is equivalent to Liouville's theorem. This is done by considering characteristic curves, which are curves along which the distribution function is advected. This turns the PDE into a set of ODEs [2].

More specifically, consider a general advection equation shown in (9).

$$\frac{\partial f}{\partial t} + \vec{c}(\vec{r})\frac{\partial f}{\partial \vec{r}} = 0 \quad (9)$$

This type of equation describes advection of the quantity $f(t, \vec{r})$ at "speed" $\vec{c}$ at "position" $\vec{r}$. To find the characteristics, we find the trajectories, $\vec{r}(t)$ for which the total derivative vanishes, shown in (10).

$$\frac{df(t, \vec{r}(t))}{dt} = \frac{df}{dt} + \frac{d\vec{r}}{dt}\frac{\partial f}{\partial \vec{r}} = 0 \quad (10)$$

The original advection equation (9) can only be satisfied if $\frac{d\vec{r}}{dt} = \vec{c}$ is satisfied. In the case of the Vlasov equation, the characteristic curves are found by solving (1)-(4). This means the value of the distribution function at any particular point can be determined by tracing the characteristic curves back until time zero. In this method, a grid is generated over $(\varphi, v_\parallel, \alpha)$. The characteristics are traced backward $(dt \rightarrow -dt)$ until time zero or until they reach the "entrance" of the interaction region.

IV. Simulation Resutls

As mentioned before, the theory predicts formation of a separatrix in phase space based on equations (5)-(6), which is illustrated in Fig. 1a (red dash line). The VL numerical model resolves in high resolution formation of the depletion in the region of phase-space known as a "phase space hole" that is shown in Fig. 1b (introducing $\zeta = \frac{\dot{\varphi}}{\sqrt{2}\omega_{tr}}$). Additionally, it is observed that dynamic frequency change can occur at the back end of an injected short pulse when particles are released from the trap. .

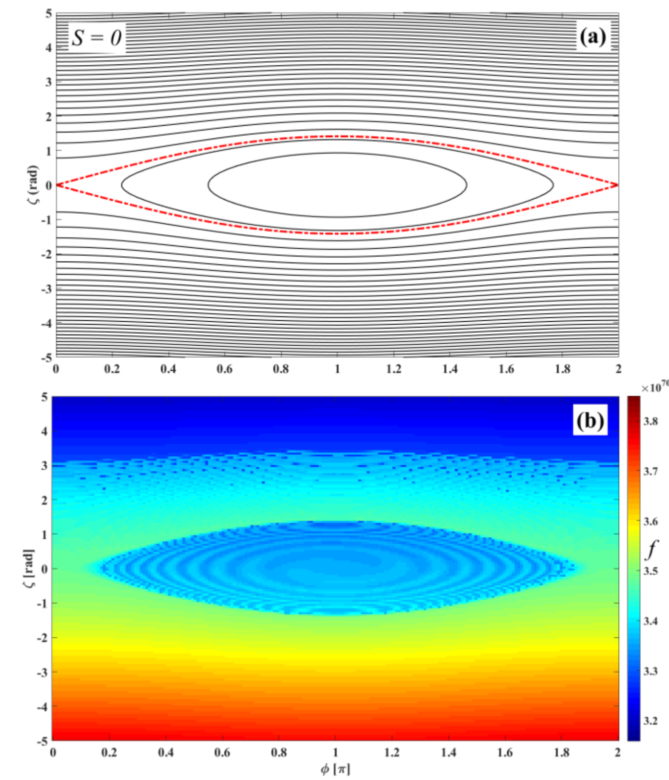

Figure 1. Formation of phase-space depletion (hole) based on a) theory (red dash line shows the separatrix), and b) numerical VL model at the equator ($S = 0$).

References

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