PARABOLIC WAVE EQUATION PROPAGATION IN A MARITIME DUCT WITH A ROUGH SEA SURFACE AND VOLUME TURBULENCE

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***Abstract*—This paper illustrates a comparison of RF X-band propagation in a marine evaporation ducting environment using the parabolic wave equation (PWE). The PWE method includes standard refractive ducting effects as well as including the effects of a stochastic rough sea surface and 3D volume turbulence scatter. The volume turbulence scatter includes spatial refractivity scales down to sub-wavelengths using a non-Markov modification to the split-step PWE.**

I. INTRODUCTION

A driving factor in tropospheric RF propagation is the spatial variability in the refractivity $N(z,r) \equiv 10^6(n-1)$ where n is the index of refraction (z is altitude, r ground range). For propagation over a curved earth, the modified refractivity $M \equiv N + 0.157z$ (z in meters). N depends on atmospheric thermodynamics, with a common form being[1] $N = 77.6(p/T)[1 + 4810e/pT]$ where p is the atmospheric pressure (mbar), T is temperature (°K), and e is the water vapor pressure (mbar). In a marine environment, vertical gradients in e near the water surface often lead to a local minimum in $M(\frac{\partial^2 M(z_d)}{\partial z^2} > 0)$ creating an evaporation duct where z_d is the duct height.

An example of 23m evaporation duct is shown in Fig. 1 derived from a radiosonde measurement during SCSMEX:

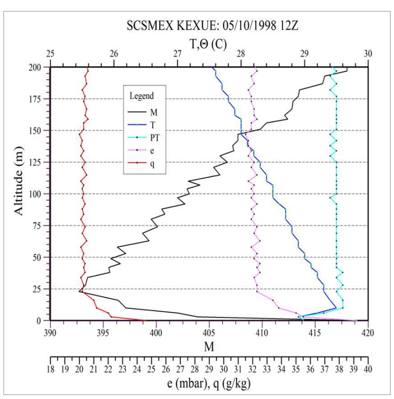

Fig. 1. 23m ED profile

Prior work([2],[3]) has considered effects of turbulence on tropospheric PWE propagation. This paper extends this to self-consistently examine the effects of a stochastic rough sea surface and small scale volume turbulence refractivity perturbations on ducted propagation at X-band using an EM parabolic wave equation model based on the split-step rotated Green's function algorithm.

II. METHODOLOGY

A brief description of how the rough sea surface and turbulent volume refractivity are computed follow.

A. Rough Sea Surface

In the PWE model, a local Fresnel boundary condition on the EM fields is applied at the air-water interface $z = \eta(x,y)$ where (x,y) are horizontal coordinates. The surface η is a stochastic realization drawn from a zero-mean random process having a spatial wavenumber spectrum S comprised from a broadband local wind-wave and narrow-band distant swell components: $S = S_{wind} + \sum_i S_{swell}$. The combined spectrum is shown in Fig. 2 for 2 swell components and a wind-wave spectrum corresponding to a wind speed $U_{10} = 10$m/s.

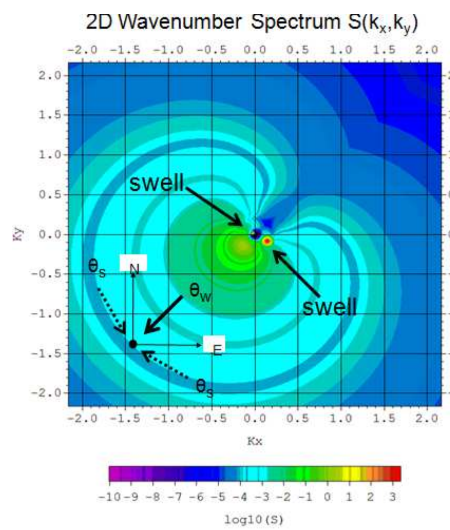

Fig. 2. 2D Wave Spectrum

A realization of η is constructed by convolution of S with a zero-mean 2D random Gaussian field G:

$$\eta(x,y) = \frac{1}{(2\pi)^2} \iint d\,k_1 d\,k_2 \sqrt{S(k_1,k_2)} G(k_1,k_2) e^{+i(xk_1+yk_2)} \quad (1)$$

The resulting η, sampled at 5cm x 5cm, is shown in Fig. 3. The sea surface significant wave height $H_{1/3} = 4\sqrt{m_0} = 0.5$m, where $m_0 \equiv \iint S\, dk_1 dk_2$ is the spectrum moment.

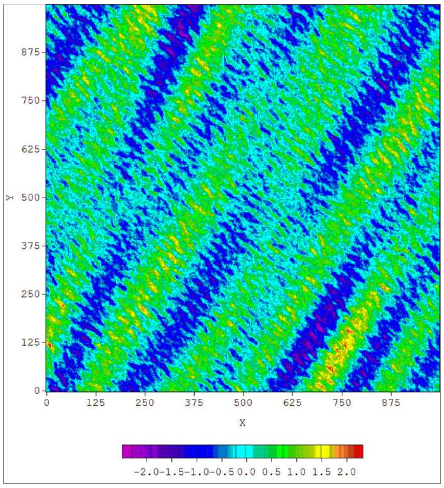

Fig. 3. Sea surface patch 1x1km

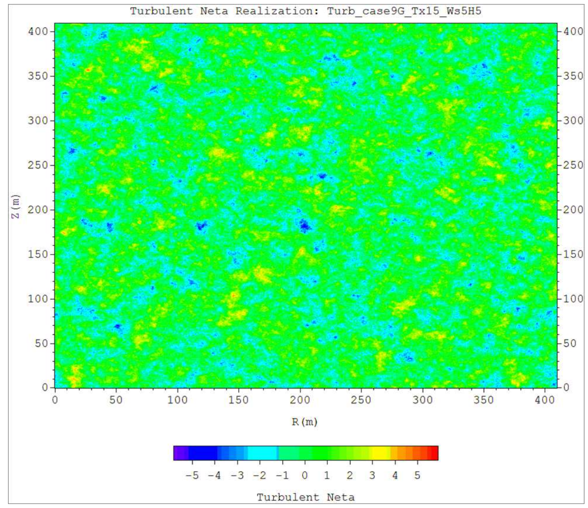

Fig. 4. Turbulent Δ N patch (N-units)

B. Turbulence Scattering

Atmospheric turbulence effects on propagation are modeled by adding a refractivity realization ΔN to the mean background refractivity: $\tilde{N} = N + \Delta N$. Realizations of ΔN are computed by a 2D convolution of a zero-mean Gaussian random field G with the 2D turbulence spectrum S weighted by the refractivity structure constant C_N:

$$\Delta N = \frac{C_n(z)}{(2\pi)^2} \iint dk_r dk_z \sqrt{S(k_z, k_r)} G(k_z, k_r) e^{+i(zk_z + rk_r)} \tag{2}$$

S is based on a modified von Kármán spectra: $S = 0.0555[K^2 + L_o^{-2}]^{-4/3} e^{-(L_0/K)^2} e^{-K^2/K_m^2} e^{-(0.5/KL_o)^2}$ with $K^2 = k_r^2 + k_z^2$, $K_m = \min(4.6/L_i, 2k_0)$ where $L_i = 0.001m$ is the inner turbulence scale length, $L_o \approx 5m$ is the outer scale length, and $k_0 = 2\pi/\lambda$ the EM wavenumber. C_N is a function of vertical gradients in the non-turbulent potential refractivity $N_v = 77.6(p/\theta)\gamma[1 + 4810\frac{\gamma q}{0.622\theta}]$, $\gamma = (p_0/p)^{0.2854}$, where $\theta = T\gamma$ is the potential temperature and q is the specific humidity. The refractivity structure constant $C_N(z)$ is: $C_N^2(z) = 2.8 \times 10^{-12} L_o^{4/3} [\frac{\partial N_v}{\partial \theta}\frac{d\theta}{dz} + \frac{\partial N_v}{\partial q}\frac{dq}{dz}]^2 / f_c$, where $f_c = 1 + e^{30(Ri/0.25-1)}$ accounts for intermittent turbulence.The gradient Richardson number $Ri = g(\frac{d\theta_v}{dz} / \frac{dU}{dz})^2$, where θ_v is the virtual potential temperature and U is the horizontal wind speed. An example of a 400×400m turbulence patch is shown in Fig. 4 corresponding to the radiosonde profile in Fig. 1.

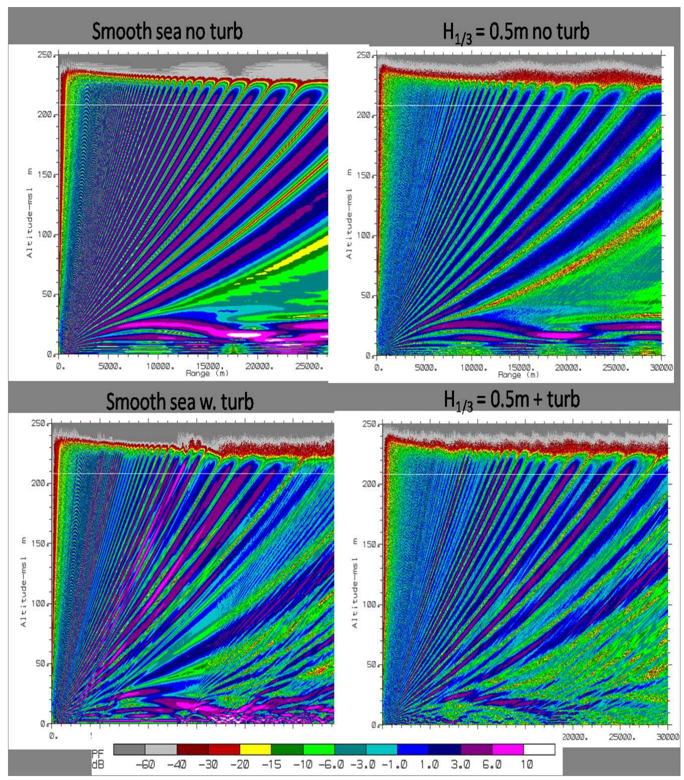

Fig. 5. Propagation results

III. Results and Conclusions

The following Fig. 5 illustrates ducted propagation in a 23m evaporation duct M-profile, for a smooth sea surface and with/without a rough sea surface and turbulence. The TX frequency is 9GHz, and height is 15m. The VTRPE electromagnetic PWE propagation model was used to compute coverage diagrams of propagation factor PF (dB). For a smooth sea and no turbulence, PWE predicts substantial energy trapping (PF $>$ 10) within the duct. Not surprisingly, a moderate rough surface and turbulence contributes to scattering out of the duct. Work supported in part by ONR CODE-331.

References

[1] B.R. Bean and E.J. Dutton, *Radio Meterology* , (Dover Publications, 1968), Chap. 3.
[2] Yung-Hsiang Chou and Jean-Fu Kiang, "Ducting and Turbulence Effects on Radio-Wave Propagation in an Atmospheric Boundary Layer," PIER B, vol.**80**, 301–325, (2014).
[3] D. Rouseff, "Simulated microwave propagation through tropospheric turbulence," IEEE Trans. Antennas Propagat., Vol. 40, No. 9, 1076-1083, (1992.)