# Wave Propagation in a Random Medium Layer with Rough Boundaries

Saba Mudaliar
Air Force Research Laboratory, Sensors Directorate
Wright-Patterson AFB, OH 45433; E-Mail saba.mudaliar@wpafb.af.mil

Wave propagation in random media, and scattering from rough surfaces have been studied extensively during the past few decades. However, the combined problem of propagation in random media with rough boundaries has not been well investigated. Yet there are many situations where there is a need for models involving random media and rough boundaries. To address this people have used hybrid approaches that directly add procedures from random media and rough surface scattering theories. Another popularly used approach for this problem is the radiative transfer theory. In contrast to these we present in this paper a unified approach which treats volumetric scattering and surface scattering on an equal footing. For illustration, we choose the following model. The permittivity of the random medium layer has a deterministic part and a randomly fluctuating part. The rough boundaries are parallel planes on the average. All random fluctuations of the problem are zero-mean stationary processes independent of each other. A point source excites waves in the random medium layer and we are interested in the propagation characteristics of waves in the layer.

Wave phenomenology in a random medium layer is adequately described by stochastic wave equations together with stochastic boundary conditions. We first translate the boundary conditions from the rough boundaries to the average planar boundaries. Using the known unperturbed Green's functions we represent the Green's function of our problem as an integral equation where the volumetric and surface fluctuations are represented in a unified manner. On averaging this equation we obtain an integral equation for the mean Green's function. We use the Weyl representation for the Green's function, and hence obtain the solution for the mean propagation constant and mean Fresnel coefficients. Noting the singularities in the Weyl integral, the mean Green's function is represented as a sum of guided modes. Although the presence of volumetric and surface fluctuations leads to coupling between modes, we observe that on the average the modes propagate in the layer without any coupling. The random fluctuations manifest themselves as increased attenuation of the coherent waves. They also lead to shifts in the location of modes. We next calculate the second moment of the Green's function for our problem as an integral equation. Using the modal representation for the Green's function we derive the second moments of the modal coefficients as a system of coupled equations. We hence learn how the fluctuations of the problem lead to coupling of the modes. Our unified approach has enabled us to identify the roles of surface and volumetric fluctuations in influencing the mode characteristics.