RESONANCES OF A SPHERICAL ANTENNA

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A spherical sheet of electric current source of radius $r=a$ is situated with its center at the origin. The surface current density is of the form: $\mathbf{J}(\mathbf{r})=\hat{\theta}J_\theta \sin\theta\,\delta(r-a)$. The amplitude of the current density at the base $\theta=\pi/2$ is given by J_θ. The electromagnetic fields for $r<a$ are obtained from a vector potential which is in the z direction and is given by (S. R. Seshadri, J. Opt. Soc. Am. vol. A25, pp. 805-810, March 2008): $A_z(\mathbf{r})=-\mu a M_N J_\theta \sin(kr)/kr$ where μ is the permeability, k is the wavenumber and $M_N=\exp(ika)\left(1+i/ka-1/k^2a^2\right)$. The fields for $r>a$ are found from another vector potential as given by $A_z(\mathbf{r})=-\mu a M_D J_\theta \exp(ikr)/kr$ where $M_D=\sin ka+\cos ka/ka-\sin ka/k^2a^2$. Only the field components $H_\phi(r,\theta)$, $E_r(r,\theta)$ and $E_\theta(r,\theta)$ do not vanish. For $r<a$ and $r>a$, the field components are determined by the use of the corresponding $A_z(\mathbf{r})$.

The complex power P_C is given by $P_C=-(1/2)\int_{V_0} d\mathbf{r}\,\mathbf{E}(\mathbf{r})\cdot\mathbf{J}^*(\mathbf{r})$ where the integration is carried out throughout the volume of the source current distribution. $\mathbf{E}(\mathbf{r})$ and $\mathbf{J}(\mathbf{r})$ are substituted and the volume integration is carried out. The base current I_θ defined by $I_\theta=2\pi a J_\theta$ is introduced. Then, P_C simplifies as $P_C=\left(\left|I_\theta\right|^2/2\right)Z_{in}$ where the input impedance is given by $Z_{in}=R_{in}+iX_{in}=(2\eta/3\pi)\left(M_D^2+iM_D N_D\right)$ and $N_D=-\cos ka+\sin ka/ka+\cos ka/k^2a^2$. R_{in} and X_{in} are investigated as functions of ka. For $M_D=0$, $X_{in}=0$ but $R_{in}=0$ also leading to the absence of radiative power (antiresonances). For $N_D=0$, $X_{in}=0$ and R_{in} is a maximum leading to a maximum of radiative power (resonances). A new feature is the occurrence of resonance for $N_D\neq 0$ and $X_{In}\neq 0$. For this resonance, $\partial X_{in}/\partial(ka)=0$ and $\partial R_{in}/\partial(ka)=0$. This new low frequency resonance and the first three normal resonances are treated in detail.

The base of the spherical antenna is excited by a voltage V. The base current $I=V/Z_{in}$ and the radiative power P_R are determined. Maximum value P_{Rm} of P_R corresponding to $X_{in}=0$ is obtained. The normalized value of the radiative power is found as $P_{RN}=P_R/P_{Rm}=M_D^2/\left(M_D^2+N_D^2\right)$. This expression is used to obtain the Q of the resonances. Finally the singular features of the dipole mode resonances of the spherical antenna are summarized.