Exact Scattering for Axial Incidence on Concave Soft and Hard Paraboloids

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A scalar plane wave propagates along the axis of a concave paraboloid of revolution. The space inside the paraboloid is filled with a linear, homogeneous and isotropic medium, and the analysis is conducted in the phasor domain with a time-dependence factor $exp\ (+j\omega t)$ and a propagation constant k. The surface of the paraboloid is either soft (Dirichlet boundary condition) or hard (Neumann boundary condition).

The analysis consists in writing the solution to the scalar wave equation as the product of the exponential function that appears in the expression of the incident plane wave times an unknown function of the coordinate that is constant on the surface of the paraboloid. An exact solution based on this type of Ansatz is possible only for two coordinate systems: the parabolic-cylinder coordinates, and the paraboloidal coordinates. The concave parabolic cylinder case was originally proposed by Lamb (1906) and recently solved by Uslenghi (2012); the unknown function in this case is a Fourier integral. For the concave paraboloid of revolution considered in the present work the unknown function is an exponential integral.

It is shown that the incident field cannot consist only of a plane wave, but must also contain an additive term that is the product of a plane wave and an exponential integral; this is due to the fact that the concave paraboloid is not a truly open surface, so that the incident plane wave is everywhere inextricably linked to its reflection at the paraboloidal surface. Similarly, the field reflected at the paraboloidal surface, after crossing the focus, consists of a plane wave propagating axially in a direction opposite to that of the incident plane wave plus a field that is the product of the reflected plane wave times an exponential integral. The incident and reflected fields are matched at the focus of the paraboloid, thus yielding exact solutions to both soft and hard boundary-value problems.