Numerical Computation of Singular Integrals in Higher Order Method of Moments Using Curved Quadrilateral Patches

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Computational techniques for analysis of antennas and scatterers based on integral-equation formulations are most frequently applied through the surface integral equation (SIE) approach, where electric and magnetic surface currents over boundary surfaces between homogeneous parts of the structure are numerically determined by the method of moments (MoM). While the MoM-SIE approach is an extremely powerful and versatile methodology for frequency-domain simulations in antenna and scattering applications, one of the most important problems and challenges in the numerical development of a MoM-SIE technique and the associated numerical discretization, implementation, and optimization for accuracy and efficiency is the treatment of singular potential two-dimensional integrals involved in MoM matrix entries with an observation (testing) point belonging to the source patch (with basis functions). This problem is even more pronounced when higher order basis functions are used for the approximation of surface currents and when such functions are defined on curved patches. In general, the techniques for dealing with singular integrals can be classified into singularity extraction (subtraction) methods and singularity cancellation or coordinate transformation methods.

This paper presents numerical computation of singular potential integrals defined on Lagrange-type generalized curved parametric quadrilateral surface elements of arbitrary geometrical orders with polynomial basis functions of arbitrary current-approximation orders. The integrals are evaluated using different methods for both the singularity extraction and the singularity cancellation. The method of extracting the singularity consists of analytical integration of a principal singular part of the integrand over a (generally not rectangular) parallelogram whose surface is close to the surface of the generalized quadrilateral near the singular point and numerical integration of the rest using Gauss-Legendre quadrature formulas. The singularity cancellation is implemented through several coordinate transformation schemes.

The integration techniques are compared in computing integrals defined on a variety of flat and curved patches, with smooth and distorted parametric geometries, respectively. The elements are varied from electrically small patches to electrically large ones, extending to a couple of wavelengths in each dimension. Basis functions are varied from constant ones to several choices of high-order polynomial bases. Integrals are computed for several different characteristic locations of the singular point on the patch or on its edge or vertex. The convergence properties of different integration procedures are evaluated in terms of the error behavior with increasing the orders of Gauss-Legendre integration formulas, i.e., numbers of integration points, over quadrilateral patches. Numerical examples demonstrate that, overall, the Duffy method for singularity cancellation is the most accurate, the most rapidly converging with the increase of the order of integration formulas, and the fastest to execute. The advantages of the Duffy method in terms of the integration accuracy and convergence rate are especially evident for some specific choices of singular points and basis functions.