New Approach for the Numerical Evaluation of Reaction Integrals in the Method of Moments

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Recently, analytical and semi-analytical evaluations of (4-D) reaction integrals involving usual linear test and source basis functions for static kernels with planar domains have been reported. Notable in this regard is the approach of (L. Knockaert, *ICEAA*, Torino, 2011) that used generalizations of the divergence theorem. Even more recently, (A. Polimeridis and J. R. Mosig, *IEEE Trans. Ant. Propagat.*, to appear) used element mappings and radial integrations to develop powerful numerical methods for handling general kernels for curved triangular source and test elements. However, the method is specifically designed only for self terms, or for edge adjacent or vertex adjacent elements, and for elements with very good aspect ratios.

In this paper we present generalizations that combine the most beneficial features of both approaches mentioned above. Specifically, the surface divergence theorem is applied twice to obtain a novel expression for the reaction between co-planar polygonal elements with kernels of singularity order $1/R$ or less, and arbitrary basis and testing functions. Interestingly, it is found that the first application of the surface divergence theorem yields a result equivalent to the radial-angular scheme, generally acknowledged as the most efficient scheme for evaluation of source integrals on planar or curvilinear elements. The second application results in a representation with two inner radial integrals plus two outer integrals over the source and element boundaries. The two radial integrals are reminiscent of the Polimeridis and Mosig scheme, but since they are applied on the original—not mapped—domains, the new scheme's performance is not degraded by elements with poor aspect ratios. Furthermore the reaction integral expression applies to general, co-planar elements and thus is not limited to self terms or to touching elements. Finally, the approach appears to be generalizable to the non-planar case as well as to kernels with singularities of higher order.

A direct numerical evaluation of the formula described is not particularly efficient, but can be made very much so by leveraging the fact that the radial and line segment interaction integrals with a static kernel can be performed in closed form. Each such closed form integral suggests a transformation that maps the integrand to a constant for $1/R$ kernels, and which therefore needs but a single Gauss quadrature sample point to be numerically exact. More realistic, dynamic kernels and varying basis and testing functions can be thought of as perturbations of the $1/R$ kernel that merely require more sampling points. Numerical results showing the efficacy of the new method will be presented.