On the Accurate Evaluation of Reaction Integrals in the Method of Moments

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The electric/magnetic field integral equations are usually solved via the method of moments (MoM), discretizing the analyzed structure with planar and/or curvilinear triangular elements. Though numerical methods for the evaluation of the reaction integrals arising in MoM discretizations have existed for over thirty years, in recent years the subject has received renewed interest in developing novel techniques achieving high accuracy, ability to treat both higher order geometry and basis representations, and efficient numerical implementation.

Early schemes for the evaluation of reaction integrals in the critical case when the source integral is (near-) singular employed the so-called “singularity subtraction” scheme. Here terms having the same asymptotic behavior as the integrand at unbounded singularities were removed from the integrand and integrated analytically, leaving a bounded difference integrand to be handled numerically. The main drawback of these schemes is their limited accuracy because the difference integrand contains higher order unbounded derivatives that are not well-approximated by polynomials, as is assumed in designing most numerical quadrature rules. Moreover, these schemes are typically either inapplicable or become increasingly complicated when treating higher order basis functions and/or curvilinear elements. The more modern approach, followed in the present study, is the “singularity cancellation” scheme in which appropriate variable transformations are applied to the integral so that the Jacobian exactly cancels the (near-) singular part of the kernel, allowing efficient numerical evaluation of source integrals using standard quadrature schemes.

Until recently, little attention has been given to the testing integral. It is fairly well-known that the source integrals, though resulting in bounded potentials, have higher order singularities near the source domain boundaries, again rendering standard quadrature schemes inefficient when test and source domain boundaries overlap or coincide. In this presentation we investigate in detail and extend the use of singularity cancellation schemes for the numerical evaluation of (near-) singular integrals, with particular attention to the final result of both surface source and testing integrations. As will be shown, the resulting proposed 4-D integration schemes also apply to curvilinear discretizations.