Homogenization Theory, Generalized Retrieval and Inverse Design for Periodic Metamaterials

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We present a general homogenization model for three-dimensional (3-D) periodic metamaterials that allows determining the effective dyadic bianisotropic parameters of periodic metamaterials in a rigorous and physically meaningful fashion. We then apply this theoretical model to an inverse design procedure aiming at realizing the desired set of effective parameters for metamaterials through the careful design of subwavelength inclusions. The homogenization theory and design process are applicable to any wave propagation directions and for arbitrary subwavelength inclusions composed of dispersive and/or nondispersive materials. This model includes two components: the theoretical homogenization of the structure at the unit cell level and at the lattice level. In the former part, we retrieve and characterize the polarizability matrices of the inclusions with their scattering signature in a two-dimensional (2-D) planar array configuration, rapidly calculating the 6-by-6 polarizability tensor of arbitrary subwavelength elements. Then, we rigorously consider the dynamic coupling among inclusions in a 3-D array and derive their dyadic characteristic equations to calculate general dispersion relations and eigenvectors for the wave propagation along the lattice. Based on our analytical approach, we are able to derive the modal properties and effective material parameters of periodic metamaterials.

We have applied our homogenization model to various metamaterial geometries, including arrays of complex inclusions such as split-ring resonators (SRR), being able to capture interesting physical mechanisms like chirality and bianisotropy and quantitatively calculate polarizability tensors and dispersion relations under normal and oblique incidence propagations. We have validated our analytical model with commercial software, showing nice agreement with results obtained from full-wave simulations of complex geometries. In our talk, we will present the complete set of dyadic effective bianisotropic parameters of the analyzed metamaterial samples, and explain their complex features with physical insights.

Finally, we will outline an efficient inverse design method for 3-D periodic metamaterials based on our homogenization theory. Since our model explicitly relates the effective parameters of metamaterials to the properties of the inclusions, we are able to determine the required polarizability dispersion to achieve desired bulk properties. In this way, we drastically simplify 3-D metamaterial design and effectively reduce it to the optimization in a 2-D space, where we perform with genetic algorithms. We believe that our homogenization model and inverse design procedure may bring fundamental physical insights into metamaterial analysis and design, making these novel media more applicable to several engineering fields.