Resonant Patch Antennas on Spherical Ground Planes and the Need for Fractional Order Associated Legendre Functions

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The traditional cavity model has been a valuable tool for the estimation of resonant modes of patch antennas since its inception nearly 30 years ago. This model is frequently used for planar antenna structures having canonical shapes in Cartesian, cylindrical, and elliptical coordinates. However, there is no limitation on the cavity model that requires the ground plane to be planar. As such, the cavity model can be useful for determining resonant modes of patch antennas having canonical shapes in spherical coordinates on a spherical ground plane. Indeed, the resonant modes readily follow by solving for the internal fields (underneath the patch) with the presumption of fictitious magnetic walls around the perimeter of the patch and electric walls on the top and bottom. Using a classical TE/TM decomposition of the fields, the resonant modes may be found.

The resonant mode solutions for such antennas immediately encounter the complexities of associated Legendre functions having fractional orders where the modes described by *"m"* and *"n"* are not necessarily integers. It is shown that the determination of *"n"* for set values of *"m"* may be graphically determined. Additional, the solution of the radial equation, giving the eigenvalue resonances, is also solved graphically for the values of *"n"* as ascertained from the solution of the theta equation.

An important consideration for the graphical solution of *"n"* is an analysis of the graphical zero crossings. Complexities arise that preclude using certain graphically obtained values that, if not treated properly, would indicate resonance modes that do not exist. The proper analysis of these values arises from a study of fractional-order associated Legendre functions as expressed in terms of Hypergeometric functions and Gamma functions.