

On a Fast Direct Solver for Source Imaging of the Brain’s Electrophysiological Activity

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Accurate imaging of the electrophysiological activity of the brain is a critical diagnostic tool for identifying and treating neurological pathologies like Parkinson’s disease and focal epilepsy. Some of the least invasive of these techniques use external measurements of the head electrical activity through electroencephalography from which they reconstruct neural signal distributions by solving an inverse problem. One critical aspect of these methods is that their capacity to extract and localize relevant biomarkers depend on an accurate representation of the electrical signal propagation through the complex head medium, typically obtained by solving numerous “forward problems”.

Improving forward modeling accuracy demands a multifaceted approach. The first critical step involves obtaining precise head anatomy and tissue properties through advanced imaging techniques like magnetic resonance imaging. Subsequently, selecting an appropriate numerical method to model the head’s electric behavior becomes crucial. While finite element and boundary element methods (BEM) are prevalent, we concentrate on BEM due to its computational efficiency and the ability to place unknowns exclusively at tissue boundaries. Among BEM formulations for head modeling, the symmetric formulation offers the highest accuracy but faces significant ill-conditioning challenges (Jan Kybic et al., 2005, *IEEE transactions on medical imaging*), particularly with dense discretizations and high tissue contrast. These limitations substantially restrict the method’s applicability to high-resolution head models. Moreover, the large number of degrees of freedom required for accurately modeling the forward problem precludes direct system inversion, necessitating iterative solvers for each forward problem solution. This becomes especially problematic in brain imaging, where forward modeling must be repeatedly solved to map external measurements to internal electrophysiological activity. The extensive number of required solutions renders the method computationally prohibitive, a challenge further exacerbated by the reduced performance of iterative solvers when confronting ill-conditioned systems.

This work addresses both challenges through innovative preconditioning strategies and the development of a novel fast-direct solver. We first apply a preconditioner that employs Calderón preconditioning principles (Simon B. Adrian et al., 2021, *IEEE Open Journal of Antennas and Propagation*), utilizing specially designed Laplacian operators to eliminate the need for dual meshes. This regularization approach effectively mitigates issues stemming from dense discretization and high-contrast ill-conditioning. Subsequently, we manipulate the system’s spectrum using the newly developed quasi-Helmholtz filters (Adrien Merlini et al., 2023, *IEEE Transactions on Antennas and Propagation*) to enhance the system matrices’ compressibility. We then compress the inverse of the spectrally-enhanced systems into skeleton forms. Leveraging Woodbury identities, we then construct a hierarchy-free fast direct solver specifically tailored for EEG-based brain electrophysiological imaging. The efficiency of the resulting schemes will be illustrated through rigorous spectral analyses in the standard 3-layered sphere model, and through extensive numerical examples of practical relevance along with timings.

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