

Fast Inversion and Uncertainty Quantification of Subsurface Electromagnetic Data Using Invertible Neural Network

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Abstract—Electromagnetic methods are essential in a range of important subsurface applications, including unexploded ordnance detection, groundwater monitoring, oil and gas exploration, and mining, among others. Inversion of electromagnetic data to subsurface model parameters remains challenging due to issues such as nonlinearity and ill-posedness. In this study, we employ an invertible neural network (INN) that creates a bijective mapping between model parameters and electromagnetic measurements, enabling efficient forward and inverse calculations. By incorporating a latent variable, the INN captures the posterior distribution of potential models, offering a natural approach for uncertainty quantification.

I. INTRODUCTION

A typical electromagnetic subsurface sensing problem can be mathematically described as $\mathbf{d} = \mathcal{F}(\mathbf{m})$, where $\mathbf{d}$ and $\mathbf{m}$ represent electromagnetic measurements and subsurface properties (e.g., conductivity, permittivity, permeability), respectively, and $\mathcal{F}$ is the forward model that links $\mathbf{m}$ and $\mathbf{d}$.

Inversion of the subsurface electromagnetic data, i.e., recovering $\mathbf{m}$ from $\mathbf{d}$ is oftentimes challenging because the mapping $\mathcal{F}$ is usually nonlinear and non-injective, making the problem ill-posed where multiple models can explain the same data. Furthermore, the direct inversion of $\mathcal{F}$ is usually infeasible. Traditional deterministic inversion solver often struggle with model ambiguity and getting stuck in local minima, providing a single solution without quantifying uncertainty. Stochastic inversion schemes offer the advantage of capturing model uncertainty and exploring multiple solutions, but they are usually computationally expensive.

In recent years, deep learning has emerged as a promising solution for subsurface inversion. For instance, convolutional and recurrent neural networks have been used for controlled-source electromagnetic inversion [1] [2], physics-driven neural networks for inversion of well logging data [3]. Despite these advancements, conventional deep learning approaches attempt to learn the ambiguous mapping to learn the inverse mapping $\mathbf{d} \rightarrow \mathbf{m}$ directly, which could lead to issues such as training instability and overfitting.

The invertible neural network (INN) address these limitation by providing a bijective mapping between model parameters $\mathbf{m}$ and responses $\mathbf{d}$ [4]. This architecture allows both forward $\mathbf{m} \rightarrow \mathbf{d}$ and inverse $\mathbf{d} \rightarrow \mathbf{m}$ mappings efficiently. The following section details the key principles behind INN, highlighting how they achieve efficient mappings, enable uncertainty quantification, and estimates the posterior distribution $\mathbf{p}(\mathbf{m}|\mathbf{d})$.

II. INN FOR ELECTROMAGNETIC INVERSION

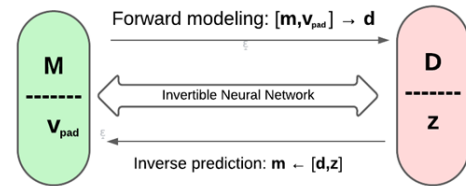


Fig. 1. Overview of the forward and inverse modeling processes using an invertible neural network. The forward propagation maps model parameters $[\mathbf{m}, \mathbf{v}_{\text{pad}}]$ to observed data $\mathbf{d}$, while the backward propagation predicts model parameters $\mathbf{m}$ given observed data and latent variables $[\mathbf{d}, \mathbf{z}]$.

The INN achieves bijectivity by using coupling layers, where the input, $\mathbf{x}$, is split into two parts $(\mathbf{x}_1, \mathbf{x}_2)$. The second part, $\mathbf{x}_2$, is transformed based on the first part, $\mathbf{x}_1$, using an arbitrarily complex function s .

$$\mathbf{y}_1 = \mathbf{x}_1, \quad \mathbf{y}_2 = \mathbf{x}_2 + s(\mathbf{x}_1) \quad (1)$$

These layers allow complex, non-linear transformations while maintaining the invertibility of the network. The INN also introduces a latent variable $\mathbf{z}$ that enables uncertainty quantification by encoding variations in $\mathbf{m}$ consistent with $\mathbf{d}$. During training, the INN learns the simpler forward mapping $\mathbf{f}(\mathbf{m}) = [\mathbf{d}, \mathbf{z}]$, and due to the invertible architecture, the INN simultaneously learns the inverse mapping, $\mathbf{m} = \mathbf{f}^{-1}(\mathbf{d}, \mathbf{z}) = \mathbf{g}(\mathbf{d}, \mathbf{z})$. The INN effectively represents the posterior $\mathbf{p}(\mathbf{m}|\mathbf{d})$ through the deterministic mapping $\mathbf{m} = \mathbf{g}(\mathbf{d}, \mathbf{z})$ [5]. Additionally, a padding vector $\mathbf{v}_{\text{pad}}$ is introduced to balance the input and output dimensions, ensuring compliance with the architectural constraints required by the network. A brief illustration for the INN's invertible structure is shown in Fig. 1. The model can then be stated as

$$[\mathbf{d}, \mathbf{z}] = \mathbf{f}(\mathbf{m}, \mathbf{v}_{\text{pad}}; \theta) \quad (2)$$

where θ is the trainable network parameters. The inverse can then be written as

$$\mathbf{m} = \mathbf{f}_m^{-1}(\mathbf{d}, \mathbf{z}; \theta), \quad \text{with } \mathbf{z} \sim p(\mathbf{z}) = \mathcal{N}(\mathbf{z}; 0, \mathbf{I}_K) \quad (3)$$

where $\mathbf{z}$ is sampled from a normal distribution and $\mathbf{m}$ is the inverted model predicted by the INN. Using the change of variable theorem, we estimate the posterior distribution

$p(\mathbf{m}|\mathbf{d})$ given the observed data using the tractable conditional probability density $q(\mathbf{m}|\mathbf{d})$ by evaluating multiple samples of the latent variable $\mathbf{z}$.

$$q(\mathbf{m} = f_m^{-1}(\mathbf{d}, \mathbf{z}; \theta) | \mathbf{d}) = |J_m|^{-1} p(\mathbf{z}) \quad (4)$$

$$J_m = \det \left(\frac{\partial f_m^{-1}(\mathbf{d}, \mathbf{z}; \theta)}{\partial [\mathbf{d}, \mathbf{z}]} \bigg|_{(\mathbf{d}, f_z(\mathbf{m}))} \right) \quad (5)$$

Each sample represents a plausible realization of $\mathbf{m}$ consistent with the observed data, collectively forming an ensemble of predictions that approximates $p(\mathbf{m}|\mathbf{d})$. The uncertainty in the predictions can be measured by analyzing the spread of the ensemble, such as calculating the variance at each point or determining percentile-based confidence intervals. These measures highlight regions of high and low confidence, with narrower intervals indicating more certain predictions and wider intervals reflecting greater uncertainty.

III. EXPERIMENTS

We tested the INN for an electromagnetic well logging data [6] inversion problem as shown in Fig. 2 upper panel. In this case, the INN is implemented using the FrEIA library, which provides a streamlined framework for constructing invertible neural networks [7]. The architecture consists of eight Real-NVP blocks, each performing invertible transformations with fully connected subnets composed of two linear layers and ReLU activations to introduce non-linearity [8]. The forward model was employed to generate 80,000 samples for training and 20,000 for testing. The INN was trained for 90 epochs, completing in 63 minutes.

The inversion process utilizes the trained INN to recover the underlying resistivity model from the EM data. For each trace, the EM input curve is preprocessed and concatenated with a randomly sampled $\mathbf{z}$ from the standard normal distribution before being input into the INN. The INN is then run in reverse to generate a subsurface resistivity model. This process is repeated 100 times to create an ensemble of resistivity models, capturing the model uncertainty. The mean of this ensemble is selected as the final predicted resistivity model. The center and lower panels of Fig. 2 show the inverted subsurface model and the uncertainties.

The time consumption for the INN to invert this model using 100 samples of latent variable $\mathbf{z}$ is 25.5 seconds. It demonstrated significantly lower prediction costs than conventional inversion methods. The execution time was 182 seconds for Occam's inversion, and it can only predict a single model without any uncertainty information. The computational costs will be even higher if the Bayesian approach, like the Markov Chain Monte Carlo (MCMC) method, is used for inversions and uncertainty quantification.

IV. CONCLUSION

This study demonstrates that INN offers a powerful framework for electromagnetic inversion by delivering both forward and inverse mappings in a single, consistent model. The ability to sample from the latent space allows straightforward

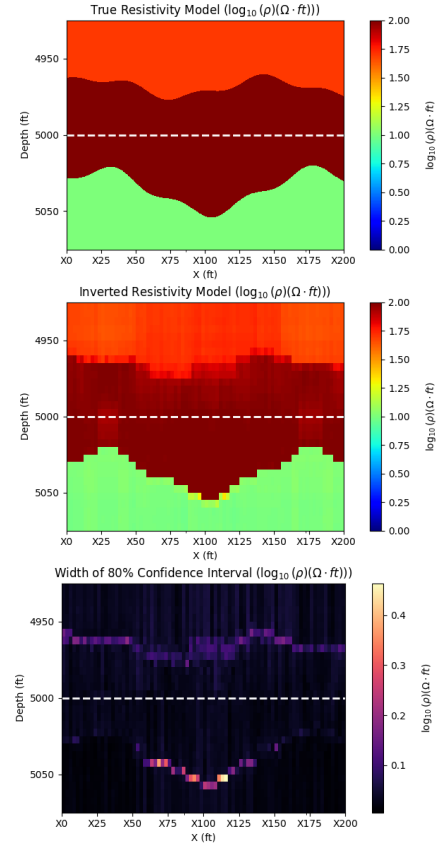


Fig. 2. An electromagnetic inversion case comparing the true model vs the inverted model produced by the INN. The width of the 80% confidence interval for each pixel is shown reflective the degree of uncertainty. The white dashed lines refers to the trajectory for data acquisition.

uncertainty quantification, addressing key limitations of conventional inversion methods. This method paves the way for fast, interpretable, and probabilistic electromagnetic inversion workflows suitable for a wide range of applications.

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