

# Invisibility Conditions in Metasurfaces and Their Relationships to Bound States in the Continuum

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A metasurface is constituted of an array of scattering particles arranged within a two-dimensional lattice. By tuning the physical parameters of both the particles and the lattice, it is possible to engineer the electromagnetic response of the metasurface. In this work, we are specifically interested in modeling and controlling the angular scattering response of a metasurface illuminated by an obliquely impinging plane wave with arbitrary polarization.

In order to rigorously model the response of a metasurface, we use the Generalized Sheet Transition Conditions (GSTCs), which provide a connection between the fields interacting with the metasurface and its intrinsic properties. A metasurface lying in the  $xy$ -plane may accordingly be modeled as a zero-thickness sheet supporting the electric and magnetic surface current densities,  $\mathbf{J}$  and  $\mathbf{K}$ , respectively. In which case, the GSTCs are given as

$$\Delta \mathbf{H}_{\parallel} = -\hat{\mathbf{z}} \times \mathbf{J}_{\parallel} + \frac{j}{\omega\mu} \nabla_{\parallel} K_z \quad \text{and} \quad \Delta \mathbf{E}_{\parallel} = \hat{\mathbf{z}} \times \mathbf{K}_{\parallel} + \frac{j}{\omega\epsilon} \nabla_{\parallel} J_z, \quad (1)$$

where  $\Delta \mathbf{H}_{\parallel}$  and  $\Delta \mathbf{E}_{\parallel}$  correspond to the difference of the tangential magnetic and electric fields between both sides of the metasurface. To connect these surface current densities to the effective material properties of the metasurface, we express them in terms of polarization densities such that  $\mathbf{J} = j\omega\mathbf{P}$  and  $\mathbf{K} = j\omega\mu\mathbf{M}$ , which are conveniently related to the effective bianisotropic susceptibilities of the metasurface.

Based on this theoretical framework, we shall see that one of the most important mechanisms for controlling the angular scattering response of a metasurface consists in tuning the frequency and angular dispersion of its scattering zeros in reflection ( $R = 0, |T| = 1$ ) and transmission ( $T = 0, |R| = 1$ ), which corresponds to the forward and backward Kerker conditions, respectively. In addition to these two conventional Kerker conditions, we study the dispersion of “invisibility” conditions, which are sometimes also referred to as lattice invisibility or lattice anapole effects. They correspond to the special cases of the forward Kerker conditions, for which the reflection coefficient is nullified ( $R = 0$ ) and the transmission coefficient is unity with zero phase shift ( $T = 1$ ).

Invisibility conditions are straightforward to derive as they correspond to the situation where the metasurface does not affect the fields, i.e.,  $\Delta \mathbf{H}_{\parallel} = \Delta \mathbf{E}_{\parallel} = 0$ . Using (1), this directly leads to the invisibility conditions given in terms of the surface currents densities by

$$\omega\epsilon\mathbf{K}_{\parallel} = \hat{\mathbf{z}} \times \mathbf{k}_{\parallel} J_z \quad \text{and} \quad \omega\mu\mathbf{J}_{\parallel} = -\hat{\mathbf{z}} \times \mathbf{k}_{\parallel} K_z, \quad (2)$$

where we have expressed the GSTCs in Fourier space and thus used the substitution  $\nabla_{\parallel} \rightarrow -jk_{\parallel}$ . At the conference, we shall see how these conditions may be further expressed in terms of the metasurface susceptibilities and how they may be extended to include multipolar contributions.

Combining both Kerker and invisibility conditions together and tuning the response of the metasurface such that these conditions overlap with each other leads to avoided crossings that naturally emerge to satisfy energy conservation since, for a lossless system, it is impossible to simultaneously have  $R = T = 0$ . We shall discuss how this mechanism is related to bound states in the continuum.