

Metasurfaces Exhibiting Strongly-Spatially-Dispersive Magneto-Electric Coupling: Misnomer or Critical Degree of Freedom?

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Spatial dispersion or *non-locality* is a characteristic of electromagnetic (EM) materials wherein the EM response (e.g., polarization) at a certain location depends not only on the local EM fields at the same location but also those at surrounding locations (Landau et al., *Electrodynamics of Continuous Media*, Oxford: Pergamon Press Ltd, 1984). This is mathematically expressed by the use of integral transforms, whose tensor kernel represents the characteristics of the spatial dispersion. This behavior is in contrast to most EM materials, e.g., ordinary dielectrics, whose EM response depends only on the local field and can be mathematically expressed by point-wise equations. (Alternatively, any spatial dispersion is negligible and can be ignored.) These latter materials can be described as being *local* or non-spatially-dispersive. The concept of spatial dispersion is simply the spatial analog of the more commonly seen concept of temporal dispersion.

When considering thin engineered materials used for the systematic transformation of EM waves, i.e., *metasurfaces*, the incorporation of spatial dispersion provides additional degrees of freedom that extend the range of possible wave transformations afforded by the metasurface. Traditionally, this spatial dispersion is assumed to be *weak*, i.e., limited to a relatively small region surrounding the point of interest. By manipulating the fundamental EM equations, a material exhibiting weak spatial dispersion can be converted into a non-spatially-dispersive material by introducing magneto-electric coupling terms (i.e., bi-anisotropy) into the now local EM response model (Caloz and Sihvola, *IEEE Antennas Propag. Mag.*, vol. 62, no. 2, 2020). These additional terms are critical degrees of freedom for the metasurface to achieve wave transformations such as perfect anomalous refraction (Ataloglou et al., *IEEE Journal of Microwaves*, vol. 1, 2021).

More recently, researchers have begun to explore the incorporation of *strong* spatial dispersion into the metasurface platform as a means to enable more exotic applications, which include speed-of-light analog computations (Cordaro et al., *Nature Nanotechnology*, vol. 18, 2023). To this end, expanded metasurface models have been sought in order to properly describe and analyze spatially-dispersive metasurfaces. Interestingly, these models often include magneto-electric coupling terms (Dugan et al., *IEEE Trans. Antennas Propag.*, vol. 71, 2023) that are themselves spatially dispersive, e.g., represented by integral transforms. On its face, such a statement seems paradoxical; indeed, if magneto-electric terms arise from an assumption of weak spatial dispersion, how can the magneto-electric terms simultaneously exhibit strong spatial dispersion? If this model is incorrect, then at best we are introducing redundant parameters that will hinder any metasurface design methods utilizing this strong spatial dispersion. Finding the answer to these questions is the principal motivation behind the work that will be presented.

In particular, we will show that a strongly dispersive magneto-electric effect is not a misnomer and can in fact represent an important degree of freedom for spatially dispersive metasurface designs. Through rigorous mathematical steps, we show how we can obtain a metasurface model that includes magneto-electric terms that are strongly dispersive. However, we also show that such an effect only occurs along certain vector directions; in other words, only certain tensor components of the magneto-electric effect are allowed to exhibit strong spatial dispersion. This understanding provides significant insight into spatially dispersive metasurface structures and solidifies the use of spatially dispersive models. The importance of these degrees of freedom will be demonstrated by analyzing certain metasurface design test cases. In addition, we will provide the necessary conditions for losslessness, passivity, and reciprocity, and we will show that the strongly dispersive magneto-electric effect provides additional degrees of freedom that can aid in the synthesis of lossless, passive, and reciprocal metasurfaces that exhibit strong spatial dispersion.