

Quantum-based Methods for solving Partial Differential Equations in Electromagnetics Engineering

E. Colella⁽¹⁾, L. Bastianelli⁽¹⁾, V. Mariani Primiani⁽¹⁾, F. Moglie⁽¹⁾, and G. Gradoni⁽²⁾

⁽¹⁾ Università Politecnica delle Marche, Marche, Italy

⁽²⁾ University of Surrey, Guildford, UK

Quantum methods for solving partial differential equations (PDEs) generally fall into a few general categories, each leveraging different quantum mechanisms to achieve potential speedups. One well-known approach is the quantum linear system solver, epitomized by the Harrow-Hassidim-Lloyd (HHL) algorithm, which uses phase estimation and eigenvalue inversion to solve large, sparse linear systems arising from discretized linear PDEs [1]. This method can yield exponential improvements over classical solvers under certain conditions (mostly when the matrix is well-conditioned and sparse). More recent developments, such as Quantum Singular Value Transformation (QSVT), provide a framework for matrix inversion in a more flexible way, often improving on the cost and error dependence of earlier solvers [2]. Time-dependent PDEs can be tackled via Hamiltonian simulations. The Trotter-Suzuki method decomposes the exponential of a Hamiltonian into products of exponentials of simpler sub-Hamiltonians, allowing a piecewise approximation of continuous dynamics [3]. Continuous-time quantum walks, which treat a PDE operator as a Hamiltonian that governs the walk, similarly simulate processes like diffusion or wave propagation on discretized grids or graphs [4]. Beyond direct simulation, variational quantum algorithms have emerged as a promising route in the noisy intermediate-scale quantum era. These approaches encode the PDE solution as a parameterized quantum state and optimize the parameters via classical feedback, thereby minimizing a cost function tied to the PDE residual or energy functional [5]. Quantum annealing or adiabatic methods, meanwhile, can solve optimization problems derived from PDE constraints by adiabatically evolving a simple initial Hamiltonian to the final problem Hamiltonian and staying in the system's ground state [3]. In our study, we implement a new parameterized quantum model, specifically designed to solve large linear systems on quantum hardware. This model is less deep and computationally faster than the proposed one in [5], reducing the complexity and speeding up convergence times. Such improvements are particularly valuable in cases where classical algorithms may fail due to the problem's size or complexity. Ongoing research is focusing on generalized quantum models that can be adapted to practical PDE problems for solving large electromagnetic problems where classical computers can fail.

References

- [1] A. W. Harrow, A. Hassidim, and S. Lloyd, "Quantum algorithm for linear systems of equations," *Physical review letters*, vol. 103, no. 15, p. 150502, 2009.
- [2] A. Gilyén, Y. Su, G. H. Low, and N. Wiebe, "Quantum singular value transformation and beyond: exponential improvements for quantum matrix arithmetics," in *Proceedings of the 51st Annual ACM SIGACT Symposium on Theory of Computing*, 2019, pp. 193–204.
- [3] M. Suzuki, "Fractal decomposition of exponential operators with applications to many-body theories and monte carlo simulations," *Physics Letters A*, vol. 146, no. 6, pp. 319–323, 1990.
- [4] A. M. Childs, "On the relationship between continuous-and discrete-time quantum walk," *Communications in Mathematical Physics*, vol. 294, pp. 581–603, 2010.
- [5] M. Lubasch, J. Joo, P. Moinier, M. Kiffner, and D. Jaksch, "Variational quantum algorithms for nonlinear problems," *Physical Review A*, vol. 101, no. 1, p. 010301, 2020.