

Sparse Bayesian Learning Based DOA Estimation Using Grid Pruning and Adaptation

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Abstract—Sparse Bayesian learning (SBL) is widely applied in direction-of-arrival (DOA) estimation. However, it is limited by complexity and grid mismatch. In this paper, an Adaptive-grid DOA estimation using grid pruning-based sparse Bayesian learning (AGGPSBL) method is proposed. To reduce complexity, grid pruning is introduced for expectation maximization (EM) framework. Furthermore, a novel adaptive-grid method is proposed for solving grid mismatch. Simulation results show that the performance of proposed AGGPSBL approaches CRB at high signal-to-noise ratios (SNR) and with lower time complexity compared to other methods.

I. INTRODUCTION

Direction-of-arrival (DOA) estimation, an essential technique in sensor array processing, has been attracted continued research interest in applications of sensor network [1]. Especially the Sparse Bayesian Learning (SBL) based DOA estimation methods have been developed in recent years. The classical SBL DOA estimation algorithms [2]–[4] use a fixed sampling grid during the iterative process of estimating. When the preset sampling grid is relatively sparse, a large deviation is caused between the estimated and the actual DOA, and it's difficult to achieve high estimation accuracy even if the off-grid correction measures exist, such as root-SBL [2], Off-grid SBL (OGSBI) [3], and Relevance Vector Machine-DOA (RVM-DOA) algorithm [4].

To address the above problem, this paper proposes an adaptive-grid DOA estimation algorithm based on grid pruning SBL (AGGPSBL).

II. SYSTEM MODEL

A. Signal Model

Consider a uniform linear array with M antennas receiving K uncorrelated narrowband sources $\mathbf{s}(t) \triangleq [s_1(t), \dots, s_K(t)]^T$ from far-field. Then, the received signal $\mathbf{y}(t) \triangleq [y_1(t), \dots, y_M(t)]^T$, at time t , is expressed as:

$$\mathbf{y}(t) = \sum_{k=1}^K \boldsymbol{\alpha}(\theta_k) s_k(t) + \mathbf{v}(t) \quad (1)$$

$$= \mathbf{A}\mathbf{s}(t) + \mathbf{v}(t), t = 1, \dots, T,$$

where $\mathbf{A} = [\boldsymbol{\alpha}(\theta_1), \dots, \boldsymbol{\alpha}(\theta_K)]$ denotes array steering matrix with $\boldsymbol{\alpha}(\theta_k) = [e^{-j2\pi d_1 \sin(\theta_k)/\lambda}, \dots, e^{-j2\pi d_M \sin(\theta_k)/\lambda}]^T$, and $\mathbf{v}(t) = [v_1(t), \dots, v_M(t)]^T$ denotes an unknown noise at time t . Based on discrete sampling in azimuth angle range, (1) is further expressed as:

$$\mathbf{y}(t) = \bar{\mathbf{A}} \bar{\mathbf{s}}(t) + \mathbf{v}(t), \quad t = 1, \dots, T, \quad (2)$$

where $\bar{\mathbf{A}} = [\boldsymbol{\alpha}(\theta_1), \dots, \boldsymbol{\alpha}(\theta_N)]$ with N denoting the number of grids, and $\bar{\mathbf{s}}(t) = [\bar{s}_1(t), \dots, \bar{s}_N(t)]^T$ is a sparse vector. By stacking $\mathbf{y}(t)$ from all snapshots:

$$\mathbf{Y} = \bar{\mathbf{A}}\mathbf{X} + \mathbf{V}, \quad (3)$$

where $\mathbf{Y} = [\mathbf{y}(1), \dots, \mathbf{y}(T)]$, $\mathbf{X} = [\bar{\mathbf{s}}(1), \dots, \bar{\mathbf{s}}(T)]$ and $\mathbf{V} = [\mathbf{v}(1), \dots, \mathbf{v}(T)]$.

III. PROPOSED ALGORITHM

A. Sparse Bayesian Learning

Under the complex Gaussian white noise assumption, the likelihood function can be denoted as:

$$p(\mathbf{Y}|\mathbf{X}; \beta_0) = \mathcal{CN}(\bar{\mathbf{A}}\mathbf{X}, \beta_0^{-1}\mathbf{I}_M), \quad (4)$$

where β_0 is noise precision and obeys Gamma distribution $p(\beta_0; a, b) = \Gamma(a, b)$ with small positive parameters a and b . Suppose $\mathbf{X}$ follows complex Gaussian distribution with zero mean, then the posterior distribution of $\mathbf{X}$ is written as:

$$p(\mathbf{X}|\mathbf{Y}; \gamma) = \int p(\mathbf{X}|\mathbf{Y}; \gamma, \beta_0) p(\beta_0; a, b) d\beta_0, \quad (5)$$

$$= \frac{\Gamma(T+a)[1 + \frac{1}{b}(\mathbf{X} - \boldsymbol{\mu})^H \boldsymbol{\Sigma}^{-1}(\mathbf{X} - \boldsymbol{\mu}) + 1]^{-(T+a)}}{\Gamma(a)(\pi b)^T |\boldsymbol{\Sigma}|}, \quad (6)$$

where $\boldsymbol{\mu} = \boldsymbol{\Gamma} \bar{\mathbf{A}}^H \boldsymbol{\Sigma}_{\mathbf{Y}}^{-1} \mathbf{Y}$, $\boldsymbol{\Sigma} = \boldsymbol{\Gamma} - \boldsymbol{\Gamma} \bar{\mathbf{A}}^H \boldsymbol{\Sigma}_{\mathbf{Y}}^{-1} \bar{\mathbf{A}} \boldsymbol{\Gamma}$, and $\boldsymbol{\Sigma}_{\mathbf{Y}} = \bar{\mathbf{A}} \boldsymbol{\Gamma} \bar{\mathbf{A}}^H + \mathbf{I}_M$. $\boldsymbol{\Gamma} = \text{diag}(\boldsymbol{\gamma})$ with $\boldsymbol{\gamma} = [\gamma_1, \dots, \gamma_N]$, and $\gamma_n, n \in \{1, \dots, N\}$ denotes the variance of n th row for $\mathbf{X}$. To estimate hyperparameter $\boldsymbol{\gamma}$, the Expectation Maximization (EM) method is used. Take the logarithm of the $p(\mathbf{Y}, \boldsymbol{\gamma})$ as objective function:

$$\mathcal{L}(\boldsymbol{\gamma}) = \ln \left[\int \int p(\mathbf{Y}|\mathbf{X}, \beta_0) p(\mathbf{X}|\boldsymbol{\gamma}, \beta_0) p(\beta_0; a, b) d\mathbf{X} d\beta_0 \right]. \quad (7)$$

Through maximizing $Q(\boldsymbol{\gamma}) = \mathbb{E}_{p(\mathbf{X}|\mathbf{Y}; \boldsymbol{\gamma})} \{\mathcal{L}(\boldsymbol{\gamma})\}$, the fixed-point theorem [5] deduces the following iterative update.

$$\gamma_n^{new} = \frac{-(MT+a) \|\boldsymbol{\mu}_n\|_2^2}{TW_n (\sum_{t=1}^T \mathbf{Y}_t^H \boldsymbol{\Sigma}_{\mathbf{Y}}^{-1} \mathbf{Y}_t + b)}, \quad (8)$$

where $W_n = -1 + \boldsymbol{\Sigma}_{nn}/\gamma_n$.

B. Grid Pruning

To reduce the computational complexity of the update for $\boldsymbol{\gamma}$, a grid pruning method is proposed, which includes grid selection, energy modification and grid deletion. Firstly, we normalize $\boldsymbol{\gamma}$ after EM method. Secondly, grids are selected where the normalized energy is less than the predefined threshold tol . Then the grids are deleted by a conservative

interval. To keep the signal energy conservation, the sum of the energy of the removed grids is equally distributed to the remaining grids. Finally, the energy of the deleted grid is set as zero.

C. Adaptive-Grid DOA Estimation

To further enhance the estimation performance by solving the grid mismatch, an adaptive-grid DOA estimation method is proposed. In each iteration for SBL, let $[\gamma_{m_1}, \dots, \gamma_{m_K}]$ denotes the K largest values in γ with $m_k \in \{1, \dots, N\}$, and $[\theta_{m_1}, \dots, \theta_{m_K}]$ is the corresponding grid points. $\mathcal{L}(\gamma)$ in (5) can be rewritten as:

$$\mathcal{L}(\gamma) = \mathcal{L}(\gamma_{m_k}, \theta_{m_k}) + \mathcal{L}(\gamma_{-m_k}, \theta_{m_k}), \quad (9)$$

$$\begin{aligned} \mathcal{L}(\gamma_{m_k}, \theta_{m_k}) = & -(MT + a) \ln[1 - (\gamma_{m_k}^{-1} + g_k)^{-1} q_k^{-1} \sum_{t=1}^T |h_{kt}|^2] \\ & - T \ln(g_k \gamma_{m_k} + 1), \end{aligned} \quad (10)$$

$$\mathcal{L}(\gamma_{-m_k}, \theta_{m_k}) = -(MT + a) \ln\left(\sum_{t=1}^T \mathbf{Y}_{-k}^H \Sigma_{-k}^{-1} \mathbf{Y}_{-k} + b\right) - T \ln|\Sigma_{-k}|, \quad (11)$$

where $\Sigma_{\mathbf{Y}} = \Sigma_{-k} + \gamma_{m_k} \alpha(\theta_{m_k}) \alpha^H(\theta_{m_k})$, $g_k = \alpha^H(\theta_{m_k}) \Sigma_{-k}^{-1} \alpha(\theta_{m_k})$, $h_{kt} = \alpha^H(\theta_{m_k}) \Sigma_{-k}^{-1} \mathbf{Y}_{-k} t$, and $q_k = \sum_{t=1}^T \mathbf{Y}_{-k}^H \Sigma_{-k}^{-1} \mathbf{Y}_{-k} t + b$. Let $\partial \mathcal{L}(\gamma_{m_k}, \theta_{m_k}) / \partial \gamma_{m_k} = 0$, we have:

$$\gamma_{m_k}^{new} = \frac{T g_k - (MT + a) q_k^{-1} \sum_{t=1}^T |h_{kt}|^2}{T g_k q_k^{-1} \sum_{t=1}^T |h_{kt}|^2 - T g_k^2}. \quad (12)$$

Then substituting (12) into (10), a more accurate DOA estimation can be obtained by:

$$\hat{\theta}_{m_k} = \arg \max_{\theta_{m_k} \in [\theta_{m_k}^l, \theta_{m_k}^r]} \mathcal{L}(\gamma_{m_k}, \theta_{m_k}). \quad (13)$$

IV. EXPERIMENT AND DISCUSSION

In this section, the proposed AGGPSBL is compared with root-SBL, RVM-DOA, OGSBI and the Cramer-Rao Bound (CRB). Considering $M=10$, $T=30$, $tol=0.1$, root mean square error (RMSE) and execution time performances are evaluated versus SNRs in different SBL DOA estimation methods.

Fig. 1 shows the RMSE changes with SNR for different SBL DOA estimation methods, respectively. The RMSE of the proposed method AGGPSBL is lower than other methods. When SNR is greater than -5dB, the RMSE of the AGGPSBL and RVM-DOA method is close to CRB. It illustrates that the DOA estimation performance of the proposed method is better than the other methods. Fig. 2 shows the execution time on 300 Monte Carlo experiment for each method versus different SNRs. The result shows that the execution time of the proposed method is lower than other methods at each SNR. It demonstrates that the proposed method achieve higher efficiency with low computational complexity.

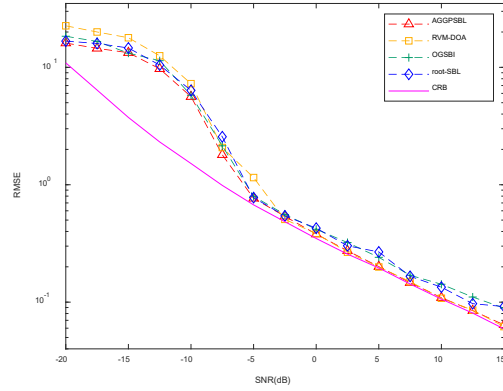


Fig. 1. The RMSE comparison of DOA methods versus different SNRs.

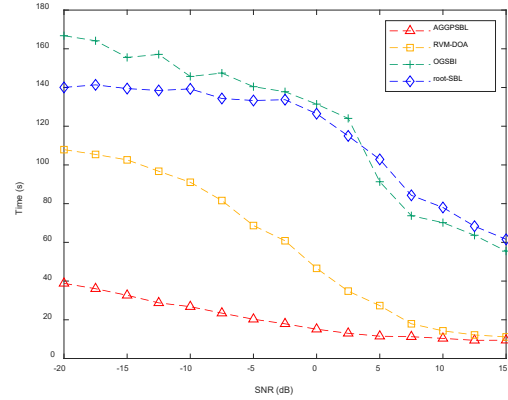


Fig. 2. The execution time comparison of DOA methods versus different SNRs.

V. CONCLUSION

This paper proposes an adaptive-grid DOA estimation algorithm based on grid pruning SBL algorithm. The proposed method is compared with some classical SBL methods in RMSE and execution time. The results show that the RMSE of the AGGPSBL is close to CRB at high SNR and the execution time of the AGGPSBL algorithm is less than other methods, which effectively proves the high-efficiency and low complexity of the proposed method.

REFERENCES

- [1] J. L. F. J. Q. Wang, H. Yu and F. Chen, "Adaptive grid refinement method for doa estimation via sparse bayesian learning", *IEEE J. Oceanic Eng.*, vol. 48, no. 3, pp. 806–819, July, 2023.
- [2] Dai J, Bao X, Xu W, Chang C., "Root sparse Bayesian learning for off-grid DOA estimation", *IEEE Signal Process. Lett.*, vol. 24, no. 1, pp. 46–50, 2017.
- [3] Yang Z, Xie L, Zhang C. "Off-grid direction of arrival estimation using sparse Bayesian inference", *IEEE Trans. Signal Process.*, vol. 61, no. 1, pp. 38–43, 2013.
- [4] Liu Z M, Huang Z T, Zhou Y Y. "An efficient maximum likelihood method for direction-of-arrival estimation via sparse Bayesian learning", *IEEE Trans. Wirel. Commun.*, vol. 11, no. 10, pp. 1–11, 2012.
- [5] Kenefic R. "Maximum likelihood estimation of the parameters of a plane wave tone at an equispaced linear array", *IEEE Trans. Acoust. Speech Signal Process.*, vol. 36, no. 1, pp.128–130, 1988.