

Addressing Distance Estimation Challenges in Underground Wireless Sensor Networks with Gamma Entropy

Nour Zaarour, Nadir Hakem, and Nahi Kandil

University of Quebec in Abitibi-Temiscamingue-LRTCS, Val-d'Or, Qc, (nour.zaarour@uqat.ca)

Abstract—Accurate distance estimation in Wireless Sensor Networks (WSNs) is crucial for localization, particularly in challenging environments like underground mines. Node deployment in such scenarios is often modeled as a Poisson Point Process with an intensity parameter (λ). However, existing methods, such as Contextual Received Signal Strength (CRSS), typically rely on prior knowledge of λ , limiting their practicality in dynamic and resource-constrained deployments. This paper introduces a novel method for estimating λ using gamma path entropy derived from network connectivity, integrated into the CRSS framework for distance estimation. Our approach achieves low normalized mean absolute error (NMAE) even in the absence of prior knowledge of λ , with performance comparable to scenarios where λ is known. By incorporating realistic path loss exponent (PLE) and noise variance values, the proposed method significantly enhances CRSS robustness, paving the way for more adaptive and scalable WSN applications in mine environments.

I. INTRODUCTION

Wireless Sensor Networks (WSNs) have become indispensable for applications that require reliable communication and monitoring in complex environments. Among these, underground mines present challenges, including harsh propagation conditions and dynamic node availability [1]. Accurate distance estimation between nodes is a cornerstone for achieving not only precise localization but also for optimizing communication protocols, managing network resources, and ensuring network resilience. In many WSN applications, node deployment is modeled as a Poisson point process (PPP), where the intensity parameter λ represents node density. This parameter plays a critical role in techniques such as Contextual Received Signal Strength (CRSS), which relies on λ to estimate inter-node distances. However, in real-world scenarios, λ is often unknown or subject to variation due to node failures or redeployment, limiting the applicability of such methods.

This paper proposes a robust method to estimate λ using gamma path entropy derived from network connectivity. By integrating this estimated λ into the CRSS framework, we enable accurate distance estimation without requiring prior knowledge of the node density. Our method is validated through simulations that reflect realistic underground mining conditions, demonstrating its ability to achieve a low normalized mean absolute error (NMAE) with minimal performance degradation compared to scenarios where λ is known.

II. SYSTEM MODEL

A linear WSN of N nodes with unknown positions is deployed in a linear WSN as a homogeneous PPP with

unknown node density λ_p . Based on multitude studies [3] we assume that the RSS in the mine environment is modeled as the sum of large-scale path loss and log-Normal shadowing as

$$RSS(d_{ij}) = P_0 - 10n \log\left(\frac{d_{ij}}{d_0}\right) + X[0, \sigma] \quad (1)$$

P_0 is the received power at the reference distance $d_0 = 1$, n is the PLE, d_{ij} is the distance between i and j , and X is the large-scale log-Normal shadowing with standard deviation σ .

A. CRSS Recall

We recall briefly the CRSS localization technique which proposes a joint estimation scheme for the range, PLE, and inter-node distances [2]. The connection status between two nodes connected at one hop is defined by C_{ij}

$$C_{ij} = \begin{cases} 1 & \text{if } RSS_{ij} \geq RSS_{thr} \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

where $j = \{1, \dots, N\}$, and $j \neq i$. RSS_{thr} is the detection threshold value. The estimated PLE, $\hat{n}$, over the entire WSN is estimated as

$$\hat{n} = \frac{-RSS_{thr} + P_0}{10 \log_{10}(\hat{R})}, \quad (3)$$

where $\hat{R}$ is the estimated range assumed to be unknown

$$\hat{R} = \frac{\hat{v}}{2\lambda_p}, \quad \hat{v} = \frac{1}{N} \sum_{i=1}^N \sum_{j=1}^N C_{ij}. \quad (4)$$

where $\bar{v}$ is the average number of adjacent neighbors per node. Each node i for $i = 1, \dots, N$, estimates its distances to its connected neighbor nodes k such as

$$\hat{d}_{ik} = 10^{\frac{P_0 - RSS_{ik}}{10\hat{n}}}. \quad (5)$$

B. Gamma Entropy for Deployment Intensity Estimation

In scenarios where nodes are deployed following a PPP with intensity λ_p , the distances between consecutive nodes are exponentially distributed with mean $1/\lambda_p$, and the path lengths (number of hops) follow a gamma distribution. For a gamma-distributed random variable X , $X \sim \Gamma(k, \lambda_\Gamma)$ the theoretical entropy is given by [4]

$$H_{theo} = k - \log(\lambda_\Gamma) + \log(\Gamma(k)) + (1 - k)\psi(k), \quad (6)$$

where k is the shape parameter corresponding to the minimum number of hops for each path, λ_Γ is the rate parameter to be estimated, and $\psi(k)$ is the digamma function.

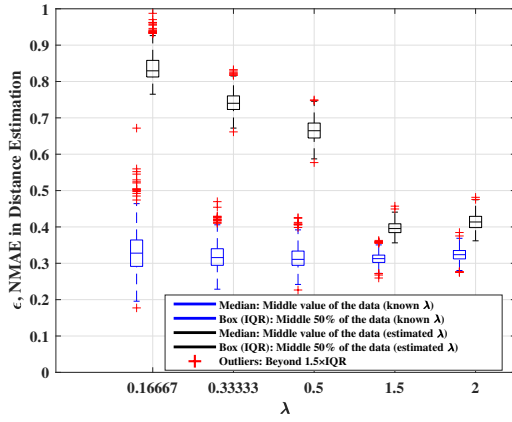


Fig. 1: Comparison of NMAE for distance estimation with known λ_p (in blue) and estimated λ_Γ (in black).

Let L_i be the path length from node i to the sink, with probability $p(L_i)$, the path length entropy is given by [5]

$$H_{emp} = - \sum_{L_i} p(L_i) \log_2 p(L_i), \quad (7)$$

We choose minimizing the entropy difference because it aligns the overall statistical behavior of the estimated process with the theoretical Poisson process. While moment matching could offer simpler estimates, entropy minimization better captures the global structure and uncertainty of the node distribution, making it more suitable for complex sensing networks, λ_Γ is hence obtained by

$$\hat{\lambda}_\Gamma = \underset{\lambda_\Gamma > 0}{\operatorname{argmin}} (H_{emp} - [k - \log \lambda_\Gamma + \log \Gamma(k) + (1-k)\psi(k)])^2 \quad (8)$$

A numerical optimization method, such as least-squares minimization, is used to find the value of λ_Γ .

III. RESULTS AND DISCUSSION

A simulation study is carried out using MATLAB over $\eta = 500$ runs. On a linear topology, N nodes are deployed on distance $d = 150m$ with different values of $\lambda = [0.16, 0.33, 0.5, 1.5, 2]$. Sink node is assumed to be at the center of the network. To emulate the mine environment, we rely on the channel characterization for underground wireless communication [3], with real-measured parameters: $n = 2.22$, $\sigma = 3.8$, $P_0 = -51.97$ dBm, and $RSS_{th} = -90$ dBm. We gauge the NMAE of estimated inter-node distances as

$$\epsilon = E_\eta \left[\frac{1}{N} \sum_{i=1}^N \frac{1}{\zeta(\phi_i)} \sum_{k \in \psi_i} \frac{|\hat{d}_{ik} - d_{ik}|}{d_{ik}} \right], \quad (9)$$

$\zeta(\phi_i)$ denotes the number of neighbors of node i , ϕ_i is the set of node i 's neighbors, and E_η is the average over η .

Fig. 1 compares the NMAE when λ is known versus when it is estimated for different values of λ . As λ increases, the NMAE decreases for both known and estimated λ . However, the error with known λ is consistently lower than the error with estimated λ . For $\lambda = 0.16$ the NMAE is approximately 0.31 with known λ compared to 0.74 with estimated λ . Conversely,

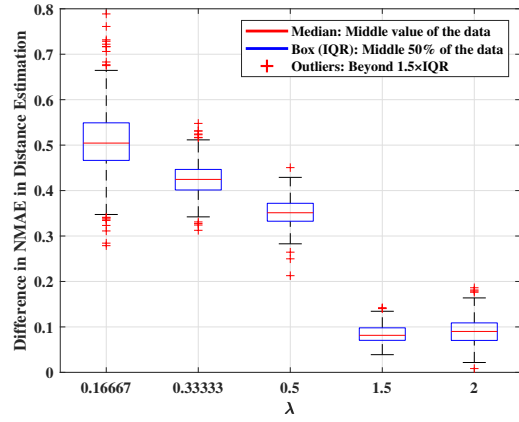


Fig. 2: Difference in NMAE for distance estimation between known and estimated λ for various values of λ .

for higher λ (e.g., $\lambda = 2$), the difference is much smaller, with NMAE of 0.42 for known λ and 0.5 for estimated λ . Fig. 2 directly highlights the difference in NMAE (d_e). For sparse deployments ($\lambda = 0.16$), $d_e = 0.43$, showing a significant impact on estimation accuracy when λ is unknown. For denser deployments ($\lambda = 2$), d_e reduces to only 0.08. In fact, in denser deployments, the proposed method shows its effectiveness by providing accurate distance estimation even when λ is unknown, with only a slight increase in error (around 8%) compared to the known λ scenario. Although the error is higher when λ is unknown, the proposed method still provides reasonable accuracy, with the loss being relatively small in denser networks. This is particularly advantageous in scenarios where direct knowledge of λ is impractical or unavailable. The ability to estimate λ extends the applicability of CRSS to more dynamic and unpredictable environments, such as underground mines, where node failures and redeployments are common.

IV. CONCLUSION

This paper presents a novel method for estimating deployment intensity λ using path entropy, enabling distance estimation in challenging environments without prior node density knowledge. Comprehensive evaluation of CRSS with known and estimated λ highlights its robustness and adaptability for dynamic underground scenarios. Future enhancements could address small-scale fading effects to improve accuracy and extend applicability to more complex environments.

REFERENCES

- [1] U. I. Minhas, I. H. Naqvi, S. Qaisar, K. Ali, S. Shahid, and M. A. Aslam, "A WSN for monitoring and event reporting in underground mine environments," *IEEE SYST J*, vol. 12, no. 1, pp. 485–496, Mar. 2018.
- [2] N. Zaarour, N. Hakem, and N. Kandil, "An accurate anchor-free contextual received signal strength approach localization in a wireless sensor network," *Sensors*, vol. 24, no. 4, 2024.
- [3] N. Hakem, G. Delisle, and Y. Coulbaly, "Radio-wave propagation into an underground mine environment at 2.4 GHz, 5.8 GHz and 60 GHz," in *EuCAP 2014*, Apr. 2014, pp. 3592–3595.
- [4] E. A. Ahmed, M. El-Morshedy, L. A. Al-Essa, and M. S. Eliwa, "Statistical inference on the entropy measures of gamma distribution under progressive censoring: EM and MCMC algorithms," *Mathematics*, vol. 11, no. 10, pp. 2298, Jan. 2023.
- [5] C. E. Shannon, "A mathematical theory of communication," *Bell System Technical Journal*, vol. 27, no. 3, pp. 379–423, July 1948.