

Quantum Modeling of Traveling-Wave Parametric Amplifiers Beyond the Stiff Pump Approximation

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Superconducting quantum computers rely on the precise distinction of quantum states through dispersive qubit readout measurements. This process involves ultra-low-power microwave signals that must be mapped to their respective qubit states with high fidelity. Consequently, a high signal-to-noise ratio is crucial. In particular, the first-stage amplifier, which needs to process input signals containing only a few tens of microwave photons, must minimize added noise. Due to their quantum-limited noise performance, superconducting traveling-wave parametric amplifiers (TWPAs) are commonly employed in qubit readout.

Modeling TWPAs, however, can be challenging as field quantization needs to be considered. A TWPA operates via nonlinear wave mixing of a weak input signal and a strong coherent pump tone. This system can be described using a Hamiltonian that captures the necessary effects. Deriving the equations of motion for the quantum mode operators from the Hamiltonian of a sufficiently complex system can be tedious. To address this, we employ an automated symbolic-numeric approach, where all necessary equations of motion for operator expectation values up to a given order are symbolically derived, and the resulting system of coupled ordinary differential equations is solved numerically.

This approach enables us to go beyond the so-called stiff pump approximation, typically employed to obtain analytical solutions. There, it is assumed that nonlinear wave mixing does not significantly deplete the energy in the pump mode, which, however, violates energy conservation. For large signals or high gain, this approximation becomes invalid. In Fig.1 (a), we validate our symbolic-numeric framework against existing numerical [S. T. Elkin et al., IEEE J. Multiscale Multiphysics Comput. Tech. **9**, 247–257 (2024)] and analytical [Y. Yuan et al., Phys. Rev. A **107**, 022612 (2023)] results. We also present the first results obtained from our symbolic-numeric approach beyond the stiff pump approximation in Fig.1 (b).

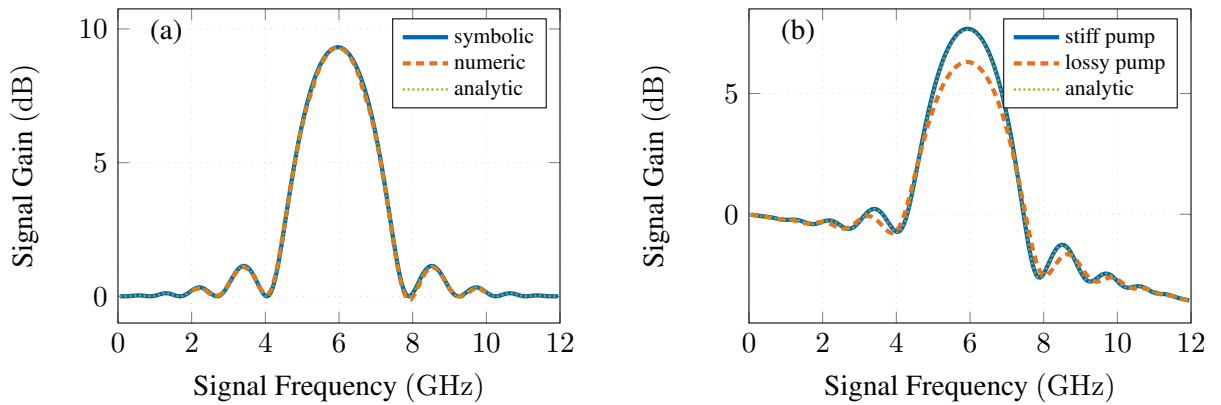


Figure 1: (a) Validation results for gain spectra of an exemplary TWPA with analytic and numeric results from the literature. (b) Gain spectra with and without the stiff pump approximation, including losses.