

# Maximum Gain Of Dielectric Dome Antennas

Lukas Jelinek

Czech Technical University in Prague, Czech Republic (lukas.jelinek@fel.cvut.cz)

**Abstract**—This work proposes a scheme based on spherical vector waves by which the maximum allowed realized gain of antenna lenses can be evaluated. The upper bound can be used to test the quality of lens design.

## I. INTRODUCTION

Antennas with high realized gain and beam scanning are commonly desired in radar applications and a typical solution to this problem will use phased array antennas which offer a wide scanning range and low profile. Unfortunately, this comes at the expense of gain degradation at high scanning angles.

To mitigate this undesirable feature, well-designed dielectric lenses can be used [1] to increase gain at a high scanning angle [2] creating a combined antenna system, often called a dielectric dome antenna, which has received considerable attention in recent years [3]–[5]. In most cases, the lenses are made of layered dielectric material, the boundary of which is specifically shaped to manipulate the wavefront and obtain the desired gain profile. The lenses are often rotationally symmetric which dramatically simplifies the design procedure [2].

In this work, we propose a numerical testbed for dielectric dome lenses to judge the performance potential of different lens designs. The procedure relies on spherical vector waves and quadratically constrained quadratic programming.

## II. SPHERICAL VECTOR WAVES

Assume a generic lens, see Fig. 1, bounded from the inside by an inscribed sphere and from the outside by a circumscribing sphere. Assuming further this lens in an otherwise empty space, then, within the inner sphere and the outside of the outer sphere, the electromagnetic field  $\mathbf{E}, \mathbf{H}$  can be expanded into spherical vector waves as

$$\begin{aligned}\mathbf{E} &= k\sqrt{Z} \sum_{\alpha} a_{\alpha} \mathbf{u}_{\alpha}^{(1)}(k\mathbf{r}) + f_{\alpha} \mathbf{u}_{\alpha}^{(4)}(k\mathbf{r}) \\ &= k\sqrt{Z} \sum_{\alpha} f_{\alpha}^{-} \mathbf{u}_{\alpha}^{(3)}(k\mathbf{r}) + f_{\alpha}^{+} \mathbf{u}_{\alpha}^{(4)}(k\mathbf{r}) \\ \mathbf{H} &= j \frac{k}{\sqrt{Z}} \sum_{\alpha} a_{\alpha} \mathbf{u}_{\alpha}^{(1)}(k\mathbf{r}) + f_{\alpha} \mathbf{u}_{\alpha}^{(4)}(k\mathbf{r}) \\ &= j \frac{k}{\sqrt{Z}} \sum_{\alpha} f_{\alpha}^{-} \mathbf{u}_{\alpha}^{(3)}(k\mathbf{r}) + f_{\alpha}^{+} \mathbf{u}_{\alpha}^{(4)}(k\mathbf{r})\end{aligned}\quad (1)$$

where  $k$  is free-space wavenumber,  $Z$  is free-space impedance,  $j$  is an imaginary unit,  $\mathbf{u}_{\alpha}^{(1)}$  represents standing waves,  $\mathbf{u}_{\alpha}^{(3)}$  represents converging waves and  $\mathbf{u}_{\alpha}^{(4)}$  represents diverging waves [6]. The relation [6]  $2\mathbf{u}_{\alpha}^{(1)}(k\mathbf{r}) = \mathbf{u}_{\alpha}^{(3)}(k\mathbf{r}) + \mathbf{u}_{\alpha}^{(4)}(k\mathbf{r})$  further allows the connecting of different sets of expansion coefficients  $\mathbf{f}^{-}, \mathbf{f}^{+}, \mathbf{f}, \mathbf{a}$  as

$$\mathbf{f}^{-} = \frac{1}{2}\mathbf{a}, \mathbf{f}^{+} = \frac{1}{2}\mathbf{a} + \mathbf{f}. \quad (2)$$

The spherical expansion is also advantageous in expressing the far field coming from the radiated waves described by the functions  $\mathbf{u}_{\alpha}^{(4)}$  which contain outgoing spherical Hankel functions with a simple approximation for  $kr \rightarrow \infty$ ,  $r$  being the distance from the coordinate center. As such, the radiated electric far field, denoted here as  $\mathbf{E}^{\infty}$ , can be written as

$$\mathbf{E}^{\infty} = \frac{e^{-jkr}}{r} \sqrt{Z} \sum_{\alpha} f_{\alpha} j^{l+2-\tau} \mathbf{Y}_{\alpha}(\hat{\mathbf{r}}), \quad (3)$$

where  $\mathbf{Y}_{\alpha}$  is a vector spherical harmonic [6],  $\hat{\mathbf{r}}$  is a unit vector in the direction of observation, index  $l$  denotes the order of the spherical wave and index  $\tau$  distinguishes TE ( $\tau = 1$ ) and TM ( $\tau = 2$ ) waves.

## III. POWER BALANCE

Assume that a set of spherical coefficients  $\mathbf{f}_{\text{in}}^{-}, \mathbf{f}_{\text{in}}^{+}, \mathbf{f}_{\text{in}}, \mathbf{a}_{\text{in}}$  exists inside the lens while a set of spherical coefficients  $\mathbf{f}_{\text{out}}^{-}, \mathbf{f}_{\text{out}}^{+}, \mathbf{f}_{\text{out}}, \mathbf{a}_{\text{out}}$  exists outside the lens. Poynting's theorem and the orthogonality of spherical waves over a spherical surface dictate [6] that

$$-P_{\text{lost}} = \frac{1}{2} \text{Re} \oint_{\partial V} (\mathbf{E} \times \mathbf{H}^{*}) \cdot d\mathbf{S} = \frac{1}{2} |\mathbf{f}^{+}|^2 - \frac{1}{2} |\mathbf{f}^{-}|^2, \quad (4)$$

where  $P_{\text{lost}}$  is the cycle mean lost power within volume  $V$ .

Relation (4) suggests that the term  $|\mathbf{f}^{+}|^2/2$  measures the cycle mean power flowing outside of the volume while the term  $|\mathbf{f}^{-}|^2/2$  is the cycle mean power flowing inside the volume. Therefore, inside the lens, the cycle mean power reflected from the lens is

$$P_{\text{refl}} = \frac{1}{2} |\mathbf{f}_{\text{in}}^{-}|^2 = \frac{1}{8} |\mathbf{a}_{\text{in}}|^2 \quad (5)$$

while the cycle mean power pumped by an antenna is

$$P_{\text{in}} = \frac{1}{2} |\mathbf{f}_{\text{in}}^{+}|^2 = P_{\text{refl}} + \frac{1}{2} (|\mathbf{f}_{\text{in}}|^2 + \text{Re} \{ \mathbf{a}_{\text{in}}^H \mathbf{f}_{\text{in}} \}). \quad (6)$$

In contrast, there are only waves propagating from the lens to infinity outside the lens and, therefore,  $\mathbf{a}_{\text{out}}, \mathbf{f}_{\text{out}}^{-} = 0$ . This allows us to express the radiated power as

$$P_{\text{rad}} = \frac{1}{2} |\mathbf{f}_{\text{out}}^{+}|^2 = \frac{1}{2} |\mathbf{f}_{\text{out}}|^2 \quad (7)$$

## IV. REALIZED GAIN

To evaluate the realized gain of a lens, the radiation intensity  $U(\hat{\mathbf{r}})$  is evaluated using (3) as

$$U(\hat{\mathbf{r}}) = \lim_{kr \rightarrow \infty} \frac{r^2 |\mathbf{E}^{\infty}|^2}{2Z} = \frac{1}{2} |\mathbf{K} \mathbf{f}_{\text{out}}|^2, \quad (8)$$

with matrix  $\mathbf{K}$  being defined as

$$\mathbf{K}_{\alpha} = j^{l+2-\tau} \mathbf{Y}_{\alpha}(\hat{\mathbf{r}}) \quad (9)$$

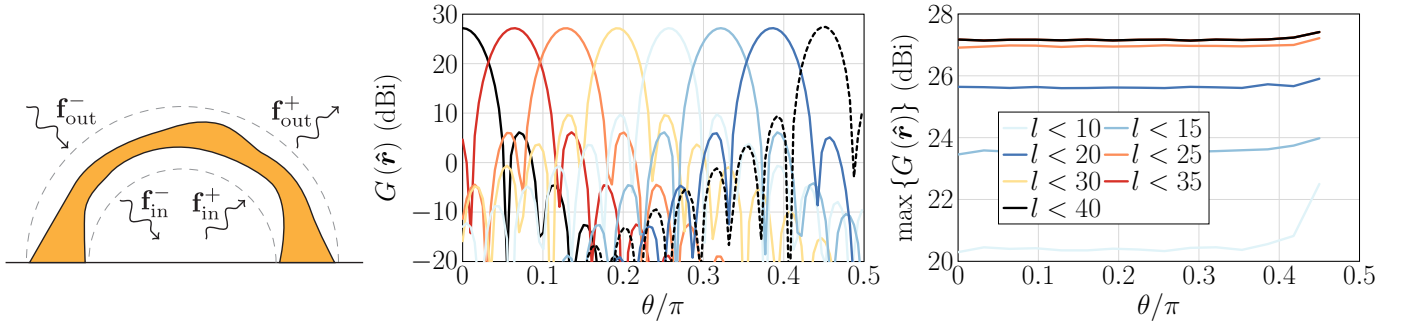


Fig. 1: (Left) Sketch of a dielectric dome antenna on a perfectly conducting plane. (Center) Maximum realized gain of a simple spherical shell of outer radius  $r \approx 4.8\lambda$  ( $\lambda$  being the vacuum wavelength) and thickness  $t = 2r/11$ . The dielectric constant is  $\varepsilon_r = 3(1 - j0.01)$ , the order of spherical waves is  $l < 40$  and the efficiency is  $\eta \approx 77\%$  in the entire plotted range. (Right) Convergence of the maximum gain with the maximum allowed order of spherical waves.

The interaction of internal and external waves with the lens is described by defining matrices  $\Psi, \Gamma$  such that

$$\mathbf{f}_{\text{out}} = \Psi \mathbf{f}_{\text{in}}, \mathbf{a}_{\text{in}} = \Gamma \mathbf{f}_{\text{in}}. \quad (10)$$

With the help of these relations, the realized gain of the antenna lens is given by

$$G(\hat{\mathbf{r}}) = \frac{4\pi U(\hat{\mathbf{r}})}{P_{\text{in}}} = \frac{4\pi \mathbf{f}_{\text{in}}^H \Psi^H \mathbf{K}^H \mathbf{K} \Psi \mathbf{f}_{\text{in}}}{\mathbf{f}_{\text{in}}^H \left( \mathbf{1} + \text{Re}\{\Gamma\} + \frac{1}{4} \Gamma^H \Gamma \right) \mathbf{f}_{\text{in}}}, \quad (11)$$

i.e., as a ratio of two bilinear forms in vector  $\mathbf{f}_{\text{in}}$  which is generated by the antenna inside the lens. The operator  $\text{Re}\{\cdot\}$  denotes the Hermitian part of the matrix.

#### A. Maximum Realized Gain

The actual form of vector  $\mathbf{f}_{\text{in}}$  depends on the antenna, its feeding, and its interaction with the lens. In all cases, however, this gain can be bounded by the maximum allowed value of (11) considering vector  $\mathbf{f}_{\text{in}}$  as a free variable. The maximum is uniquely given by the maximum eigenvalue  $\max\{\lambda\}$  of the following generalized eigenvalue problem

$$4\pi \Psi^H \mathbf{K}^H \mathbf{K} \Psi \mathbf{f}_{\text{in}} = \lambda \left( \mathbf{1} + \text{Re}\{\Gamma\} + \frac{1}{4} \Gamma^H \Gamma \right) \mathbf{f}_{\text{in}}. \quad (12)$$

When evaluated at different observation angles, this upper bound can be used as a benchmark of lens performance.

#### V. EXAMPLE

At the symposium, it will be shown that for vectors  $\mathbf{f}_{\text{in}}$  generated by real antennas in the lens, the gain predicted by (11) perfectly matches the result of commercial electromagnetic simulators, thus validating the theory presented above. In this abstract, we only show the upper bound on lens gain. For the sake of simplicity, we chose a simple dielectric spherical shell of thickness  $t$  as an example, see Fig. 1. The matrices  $\Psi, \Gamma$  are, in this case, obtained analytically, which also allows to study numerical aspects of the evaluation. Particularly, the extremely high dynamic range of matrices  $\Psi, \Gamma$ , which needs to be treated carefully when working in finite precision arithmetic.

Not all spherical waves are used since the lens is placed on top of a ground plane. Placing the ground plane in the  $xy$  plane and denoting  $m$  the azimuthal number, only TE spherical waves with even  $l+m$  and TM spherical waves with odd  $l+m$  remain. This removes half of the allowed degrees of freedom.

The maximum allowed realized gain for the spherical cover is shown in Fig. 1 for selected electric size and various directions  $\hat{\mathbf{r}}$  along which the gain is maximized. As expected, the spherical cover offers equalized gains in all directions. The gain is limited by losses and reflections from the lens, which effectively limits the order of spherical waves involved in this fundamental bound, as can be seen from the rightmost panel of Fig. 1. Conservative estimate of necessary orders of spherical waves is given by  $l < kr + (kr)^{1/3} + 3$ .

More details are to be given at the symposium, including a careful validation of the method, details of  $\Psi, \Gamma$  acquisition, matrix-free solution to (12) and examples of realistic lenses.

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