

Hierarchical Implementation of Operator-Adapted Wavelet Decomposition Using Arbitrary Polygons for Multiscale FEM Problems

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Efficient numerical analysis of electromagnetic problems containing multiscale features presents a significant challenge. Although methods such as h -refinement, which are popular in the finite element method (FEM) literature, eliminate the need to re-mesh the entire domain, they often suffer from coupling between different scale levels. Wavelet-based approaches seek to achieve scale-decoupled systems but frequently encounter high computational costs and difficulties in generalizing to unstructured grids and diverse problem types. We have developed a sparse linear algebra-based operator-adapted wavelet decomposition approach that provides an accurate, efficient, and generalizable solution to these challenges. A nearly linear computational complexity is obtained using a coarsening procedure built upon sparse linear algebra-based hierarchical equations. By employing sparse matrix-vector multiplications, the entire algorithm is constructed hierarchically from the finest to coarser levels. Although this algorithm follows a coarsening procedure, our previous mesh generation strategies used a subdivision-based approach to simplify mesh hierarchy implementation.

In this study, we investigate the operator-adapted wavelet decomposition method applied adaptively using a mesh hierarchy derived from a coarsening procedure spanning the finest to coarser levels. At the finest level, the meshes consist of triangular elements, which are geometrically coarsened at each step to form *polygonal* meshes composed of quadrilaterals, pentagons, hexagons, etc. During the usual FEM steps, Whitney one-form edge basis functions are assigned to these polygons, enabling the construction of operator-agnostic refinement matrices. Smooth regions can be covered with a coarse mesh based on fewer polygon elements, while high-resolution areas are captured using more polygons and, if necessary, triangular elements at the finest level. The proposed operator-adapted wavelet decomposition method yields an efficient and accurate hierarchical FEM solution in a complex domain with multiscale features. In the presentation, we will illustrate the application of the methods in several challenging multiscale problems.