

# Comparing Limitations of MAMPs and Holland-Simpson Thin Wire Models to Develop a Unified Wire Formalism in FDTD

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The finite-difference time-domain (FDTD) method is a flexible and powerful algorithm for modeling electromagnetic behavior. Applying discretized Maxwell's equations on a Cartesian grid, the method can obtain solutions up to second-order accuracy. Despite its many strengths, FDTD is unable to resolve geometrically small structures whose solutions exceed second-order complexity. A common example is a wire with a radius smaller than a single grid cell. To address this limitation, thin wire models have been developed over the years to facilitate the incorporation of wires into FDTD simulations.

Historically, two prevalent formulations for thin wire models have been developed. The Holland and Simpson model stores current and charge density on the wire using an auxiliary 1D grid that couples with FDTD field components adjacent to the wire. Conversely, the Umashankar model employs quasi-static approximations to modify the update equation for adjacent field components. Although various improvements have been made to either model, no study has compared their performance. In this report, we investigate the differences between the Holland-Simpson and Modified Assigned Material Parameters (MAMPs) models, which is used in place of the Umashankar model as its latest enhancement.

Although the effectiveness of wire models are classically verified by implementation in an antenna system, this study instead measures the scattering field solution of a wire when excited by an incident TMz-polarized plane wave. This approach is chosen to isolate the wire model from potential errors introduced by other parts of the system. Furthermore, wire ends have also been shown to produce significant errors due to charge accumulation. To eliminate this factor, an axially symmetric test setup of an infinite wire is selected. The field is then measured at all points within two wavelengths of the wire and compared against an analytical solution. Both MAMPs and Holland-Simpson models are verified for the case of a very thin wire ( $r = 0.001\Delta x$ ) and relatively thick wire ( $r = 0.4\Delta x$ ). Numerical results reveal that MAMPs produces relatively accurate results for thick wires, but introduces an artificial scattering for thin wires. On the other hand, the Holland-Simpson model produces accurate solutions for thin wires, but is unstable for thick wires. Moreover, applying a reduction in time step to enforce stability does not result in a correct solution.

By re-conceptualizing the interpretation of MAMPs, we show that MAMPs effectively uses modified adjacent field components as four separate auxiliary grids to store wire current and charge density. However, this causes incident fields at these locations to be overwritten, effectively enforcing a perfect conductor that results in artificial scattering. Further, the analytical solution of plane wave scattering reveals that the field can be decomposed into modes corresponding to tangential phase variation of induced wire surface current. While the solution is primarily dominated by the zeroth-order mode corresponding to no phase variation, higher-order modes become non-negligible as wire radius increases. This reveals why the Holland-Simpson model cannot produce accurate solutions for thick wires; since only one auxiliary grid is used, only the zeroth-order mode can be represented.

Building on insights from our investigation, we present a new unified wire model capable of accurately representing both thick and thin wires without introducing artificial scattering. Additionally, a phase adjustment technique is applied to correct phase errors introduced from the quasi-static approximation. Under the symmetric test setup, the new model demonstrates stability for all wires with radius  $r < 0.8\Delta x$ . At a resolution of 20 cells per wavelength, the model achieves an amplitude error within 5% and phase error within  $5^\circ$  across the entire simulation domain when compared to the analytical solution.