

A time-domain FEM-BEM symmetric coupling for transient electromagnetic scattering

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Finite element - boundary element (FEM-BEM) coupling is a common approach to model electromagnetic scattering by inhomogeneous bodies. When material coefficients of the scatterer also vary in time or excitation is broadband, a time-domain FEM-BEM coupling scheme is appropriate. In this talk, we investigate a symmetric coupling of finite elements and boundary elements, which was thoroughly studied in (R. Hiptmair, SIAM J. Numer. Anal, 41(3), 919-944, 2003) for frequency-domain. In the time-domain regime, the interior volume electric field $e(\mathbf{r}, t)$ and the exterior boundary electric current $\mathbf{j}(\mathbf{r}, t)$ are the solution to the following strongly coupled system

$$\begin{aligned} q(\boldsymbol{\varphi}, e) - \frac{1}{\eta} \left(\mathbf{n} \times (\mathbf{n} \times \boldsymbol{\varphi}), \dot{\mathcal{T}}(\mathbf{n} \times e) \right)_{\Gamma} + \left(\mathbf{n} \times (\mathbf{n} \times \boldsymbol{\varphi}), \left(\frac{1}{2} \dot{\mathcal{I}} + \dot{\mathcal{K}} \right) \mathbf{j} \right)_{\Gamma} &= - \left(\mathbf{n} \times \boldsymbol{\varphi}, \dot{\mathbf{h}}^{inc} \right)_{\Gamma}, \\ \left(\mathbf{n} \times \boldsymbol{\lambda}, \left(-\frac{1}{2} \dot{\mathcal{I}} + \dot{\mathcal{K}} \right) (\mathbf{n} \times e) \right)_{\Gamma} + \eta \left(\mathbf{n} \times \boldsymbol{\lambda}, \dot{\mathcal{T}} \mathbf{j} \right)_{\Gamma} &= - \left(\mathbf{n} \times \boldsymbol{\lambda}, \dot{\mathbf{e}}^{inc} \right)_{\Gamma}, \end{aligned}$$

where $\dot{\mathcal{T}}$, $\dot{\mathcal{K}}$ and $\dot{\mathcal{I}}$ are the time-differentiated version of the electric field integral operator, the magnetic field integral operator and the identity operator, respectively. The FEM weak form is given by

$$q(\boldsymbol{\varphi}, e) = (\nabla \times \boldsymbol{\varphi}, \mu^{-1} \nabla \times e)_{\Omega} + (\boldsymbol{\varphi}, \sigma \dot{e})_{\Omega} + (\boldsymbol{\varphi}, \epsilon \ddot{e})_{\Omega}.$$

Spatial discretization relies on Nedelec **curl**-conforming finite elements (edge elements) and Raviart-Thomas (or Rao-Wilton-Glisson) div_{Γ} -conforming boundary elements. Temporal discretization, on the other hand, is based on the collocation-in-time scheme. More specifically, both unknowns are temporally expanded in the set of shifted piecewise-quadratic Lagrange functions, then the semi-discrete coupled system is tested with the Dirac delta distributions. The resulting matrix system preserves the sparseness of finite element matrices and the symmetry of the boundary integral Calderón projection operator. It is effectively solved by the marching-on-in-time scheme.

Numerical results demonstrate the capability and accuracy of the proposed time-domain FEM-BEM coupling in modeling electromagnetic scattering by objects with complex material properties. In addition, they reveal that the symmetric coupling scheme seems to be resonant-free and late-time stable, especially in the presence of electric conductivity σ .