

Developments in the Generalized-Lorenz Gauged Potential-Based Finite Element Method

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The finite element method (FEM) is one of the most popular and powerful approaches for evaluating electromagnetic fields in complex geometries. Typical FEM implementations discretize the vector wave equation for the electric field, leading many important developments to be made in this framework. However, this approach suffers from low-frequency breakdown, meaning the system becomes increasingly ill-conditioned as frequency is reduced. A popular solution to this problem is tree-cotree splitting (TCS), which decomposes the field into a set of pure gradient and curl-conforming basis functions to represent its irrotational and solenoidal components respectively (J.- M. Jin, “The Finite Element Method in Electromagnetics,” Wiley, 2014). While this modified system does not exhibit low-frequency breakdown, it can suffer from poor performance in situations where the original formulation is effective, such as at high frequencies. This issue becomes critical when modeling highly multiscale geometries as the number of elements per wavelength varies substantially throughout the mesh. To alleviate this problem, a method discretizing equations for the electromagnetic potentials instead of the fields has been proposed (Y. -L. Li *et al.*, “Finite Element Implementation of the Generalized-Lorenz Gauged $A\text{-}\Phi$ Formulation for Low-Frequency Circuit Modeling,” *IEEE Transactions on Antennas and Propagation*, vol. 64, no. 10, pp. 4355-4364, 2016). Due to use of the generalized-Lorenz gauge, this system also does not experience low-frequency breakdown, and has the potential to alleviate some of the issues of TCS at higher frequencies. However, the method currently lacks many of the important features which have been developed for the field-based formulation. Here, we develop the necessary theory to incorporate lumped ports and absorbing boundary conditions (ABCs) into the potential-based method. In Fig. 1, we show strong agreement between the potential- and field-based formulations simulating a coplanar waveguide resonator using lumped ports, and demonstrate that the potential-based formulation does not experience low-frequency breakdown. Further, we extend the potential-based method with lumped ports and ABCs to the time domain to enable applications to broadband and nonlinear problems.

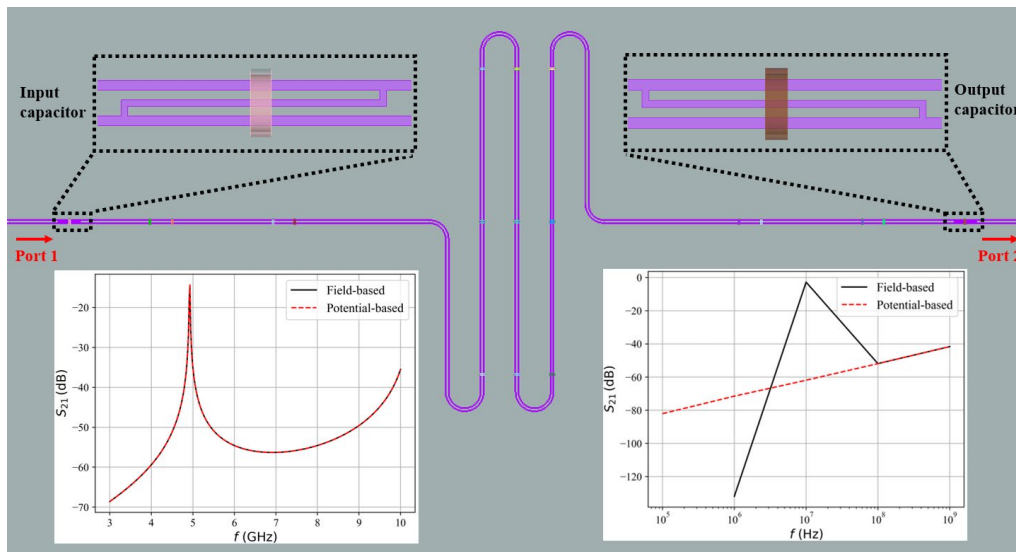


Figure 1. A coplanar waveguide resonator capacitively coupled to 50 Ω ports. The results show agreement between the methods at high frequencies, and stability of the potential-based method at low frequencies.