

Mean Green's Function of a Random Medium with Inhomogeneous Background

Saba Mudaliar

Sensors Directorate, Air Force Research Laboratory, WPAFB Ohio, USA

Although waves in random media have been extensively studied during the past several decades not much has been reported on multiple scattering in random medium with varying background. In this paper we consider the problem of waves in random medium with monotonically varying background permittivity along an axis. The random fluctuations of permittivity normalized with respect to that of the background is statistically homogeneous. We formulate the Green's function of this problem as an integral equation involving background Green's function (BGF) and the random permittivity. We construct the BGF using the Weyl representation, and ensure that it satisfies continuity, step discontinuity of its directional derivative along stratification axis, and radiation conditions. Note that the monotonic variation of permittivity along the stratification axis leads to total internal reflection of the waves at some point along the stratification axis whose location depends on the transverse wave number. Our BGF properly takes into account the singularity that originates from this physical phenomenon. Unlike the corresponding Green's function with uniform background our BGF is asymmetric as expected.

Multiple scattering theory leads us to the Dyson equation that governs the mean Green's function (MGF) for our problem. Based on the assumption of small random fluctuations we employ the Kraichnan approximation to the mass operator. Note that this is a nonlinear equation for MGF for which we seek a perturbations solution. We find that the first order term introduces resonant excitation to the unperturbed Helmholtz operator, which in turn results in the appearance of secular terms. On observing that the scale of variation of amplitude and phase of the wave function are well separated we employ a two-scale analysis and seek the leading order solution for our MGF. The condition for the elimination of the secular terms leads to two coupled nonlinear equations for the amplitude function of up and down-going waves. We observe that the product of the amplitude functions is invariant on the large-scale parameter. This decouples the two equations. A transformation further converts the nonlinear differential equation to a linear second order differential equation with constant coefficients that can be readily solved. Thus, we arrive at the solution for the large-scale coefficients, and hence construct the desired MGF. As expected, multiple scattering process induces attenuation of the coherent waves. There are several features that stand out in contrast to the MGF with homogeneous background permittivity. First is its asymmetric structure along the direction of stratification. In addition to the phase, multiple scattering also manifests itself in the amplitude of the coherent waves especially in the region close to the source. The correction to the phase of the coherent waves depends on the coordinate of the axis of stratification even though the normalized permittivity fluctuation is assumed to be statistically homogeneous. Several other special features of our results will be elaborated at the conference.