

Analytical Average Backscattered Range Profiles for an Object Above a Rough Surface

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Abstract—The coherent radar cross-section range profile for an object above a randomly rough surface is analytically derived. This is done by taking a four-path model approximation to simplify the many interactions that may occur between the object and the surface. A decoupled method is employed to express these four paths in terms of the known analytical expressions of surface scattering from the small-slope-approximation. Results for this method are shown from a PEC plate suspended above a rough surface.

I. INTRODUCTION

Modeling the range profile of a scattering object is of interest in many fields. For objects that exist in the presence of a rough surface, the resulting scattered fields involve interactions with the rough surface. Given the interactions between target and surface, the total scattered field of this joint problem also must be solved statistically. This work presents an analytical formulation for the coherent RCS range profile of an object above a rough surface as an extension of a previously developed frequency domain solution [1].

II. TIME DOMAIN SCATTERED FIELD

The scattered field in time is related to the scattered field in the frequency domain by an inverse Fourier transform, $\mathbf{F}^{-1}$. The scattered field of an object in the presence of a rough surface is a summation of two terms

$$\bar{E}_s(t) = \langle \bar{E}_s(t) \rangle + \bar{E}_s^f(t). \quad (1)$$

The mean scattered field, $\langle \bar{E}_s(t) \rangle$, characterizes the “coherent” field whose phase remains predictable as surface roughness is varied, while the fluctuating part, $\bar{E}_s^f(t)$, is an incoherent field whose phase varies randomly as surface roughness changes. Here we discuss the coherent response; the incoherent field has also been derived and will be discussed in the conference presentation.

III. ROUGH SURFACE SCATTERING

Consider a plane wave propagating in direction $\bar{k}_{i-}$ (where the sign in the subscript denotes its downward path) with incident angle θ_i , illuminating a rough surface. The field scattered from the rough surface in the absence of the target from the first-order small-slope-approximation (SSA1) is given by [2]

$$\bar{E}_r(\bar{r}) = \int_{\Omega} d\bar{k}_{\perp} \left(\bar{F}(\bar{k}_{+}, \bar{k}_{i-}) \cdot \bar{E}_0 \right) e^{-j\bar{k}_{+} \cdot \bar{r}} \quad (2)$$

where the region of integration, Ω , excludes evanescent waves and defines a circular region given by $k_x^2 + k_y^2 < k^2$, and the 2 by 2 scattering matrix, $\bar{F}$, is a function of the random surface profile [2]. The average of (2) (i.e. the rough surface coherent field) is

$$\langle \bar{E}_r(\bar{r}) \rangle = \bar{\Gamma}_{SSA}(\theta_i) \cdot \bar{E}_0 e^{-j\bar{k}_{i+} \cdot \bar{r}} \quad (3)$$

which is a well known solution [3] where the average scattered field from a rough surface exists only in the specular scattering direction, $\bar{k}_{i+}$, and can be interpreted as scattering from a flat surface. The effective reflection coefficient of the surface is given by $\bar{\Gamma}_{SSA}(\theta_i)$ which is the multiplication of the standard Fresnel coefficient and the coherent surface reduction coefficient, $\exp -2(hk \cos \theta_i)^2$. Here h is the rms height of the surface and k is the wavenumber. Note that as the surface roughness increases with respect to wavelength, the coherent reflection coefficient exponentially attenuates.

IV. DECOUPLED METHOD FOR AN OBJECT ABOVE ROUGH SURFACE

Consider now the joint problem of a target above the rough surface. A four-path model approximation can be used that limits the interactions between the surface and target to those shown in Fig. 1. Using the ‘decoupled’ method (see [1]), each of these four-paths can be expressed in terms of the independent scattering from the surface as in (2) and the scattering amplitude of the target given by $\bar{S}$.

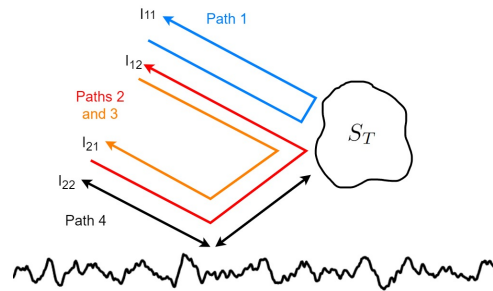


Fig. 1. Four-path model for backscatter.

The total scattered far-field of this joint problem for a plane wave of a given frequency is given as the summation of the four paths

$$\bar{E}_s(\omega) = \frac{e^{-jkr}}{r} [I_{11}(\omega) + I_{12}(\omega) + I_{21}(\omega) + I_{22}(\omega)] \quad (4)$$

The paths are expressed for the backscatter case as

$$I_{11}(\omega) = \hat{e}_s \cdot \bar{S}(-\bar{k}_{i-}, \bar{k}_{i-}) \cdot \bar{E}_0 \quad (5)$$

$$I_{12}(\omega) = \int_{\Omega'} d\bar{k}'_{\perp} \hat{e}_s \cdot \bar{S}(-\bar{k}_{i-}, \bar{k}'_{+}) \cdot \bar{F}(\bar{k}'_{+}, \bar{k}_{i-}) \cdot \bar{E}_0 \quad (6)$$

$$I_{21}(\omega) = \int_{\Omega''} d\bar{k}''_{\perp} \hat{e}_s \cdot \bar{F}^{tx}(\bar{k}''_{+}, \bar{k}_{i-}) \cdot \bar{S}(-\bar{k}''_{+}, \bar{k}_{i-}) \cdot \bar{E}_0 \quad (7)$$

$$I_{22}(\omega) = \int_{\Omega''} \int_{\Omega'} d\bar{k}''_{\perp} d\bar{k}'_{\perp} \hat{e}_s \cdot \bar{F}^{tx}(\bar{k}''_{+}, \bar{k}_{i-}) \cdot \bar{S}(-\bar{k}''_{+}, \bar{k}'_{+}) \cdot \bar{F}(\bar{k}'_{+}, \bar{k}_{i-}) \cdot \bar{E}_0 \quad (8)$$

where each interaction of the surface includes an integration in the spectral domain as in (2). Note that the operator superscript tx represents a time reciprocity relationship where the cross-polarization terms in the 2 by 2 matrix are negated and interchanged.

Applying (4) for multiple frequencies, the total scattered field can be represented in the time domain as

$$\bar{E}_s(t) = \frac{1}{r} [\mathbf{F}^{-1} [I_{11}(\omega)] + \mathbf{F}^{-1} [I_{12}(\omega)] + \mathbf{F}^{-1} [I_{21}(\omega)] + \mathbf{F}^{-1} [I_{22}(\omega)]] \quad (9)$$

$$= \frac{1}{r} [I_{11}(t) + I_{12}(t) + I_{21}(t) + I_{22}(t)]. \quad (10)$$

A. Coherent Average

The ensemble average of the total scattered field in the time domain given in (9) is found given by the individual averages of the paths.

$$|\langle \bar{E}_s(t) \rangle|^2 = \frac{1}{r^2} |I_{11}(t) + \langle I_{12}(t) \rangle + \langle I_{21}(t) \rangle + \langle I_{22}(t) \rangle|^2 \quad (11)$$

Note that I_{11} is not affected by the average given the deterministic nature of the object backscatter it represents. Given the linearity of the inverse Fourier transform, these averages can be solved in the frequency domain as detailed in [1] and shown to be

$$I_{11}(t) = \mathbf{F}^{-1} [\hat{e}_s \cdot \bar{S}(-\bar{k}_{i-}, \bar{k}_{i-}) \cdot \bar{E}_0] \quad (12)$$

$$\langle I_{12}(t) \rangle = \mathbf{F}^{-1} [\hat{e}_s \cdot \bar{S}(-\bar{k}_{i-}, \bar{k}_{i+}) \cdot \bar{\Gamma}_{SSA}(\theta_i) \cdot \bar{E}_0] \quad (13)$$

$$\langle I_{21}(t) \rangle = \mathbf{F}^{-1} [\hat{e}_s \cdot \bar{\Gamma}_{SSA}(\theta_i) \cdot \bar{S}(-\bar{k}_{i+}, \bar{k}_{i-}) \cdot \bar{E}_0] \quad (14)$$

$$\langle I_{22}(t) \rangle = \mathbf{F}^{-1} [\hat{e}_s \cdot \bar{\Gamma}_{SSA}(\theta_i) \cdot \bar{S}(-\bar{k}_{i+}, \bar{k}_{i+}) \cdot \bar{E}_0] + \mathbf{F}^{-1} [\langle I_{22}^{cc}(\omega) \rangle] \quad (15)$$

$$\cdot \bar{\Gamma}_{SSA}(\theta_i) \cdot \bar{E}_0] + \mathbf{F}^{-1} [\langle I_{22}^{cc}(\omega) \rangle] \quad (16)$$

where $I_{22}^{cc}(\omega)$ is the coherent correction term of the I_{22} path arising from second order statistics [1]. Besides this term, the coherent solution can be interpreted as a target above a flat surface with an effective reflection coefficient as shown in (3) which attenuates to zero as surface roughness increase.

V. EXAMPLE PROBLEM

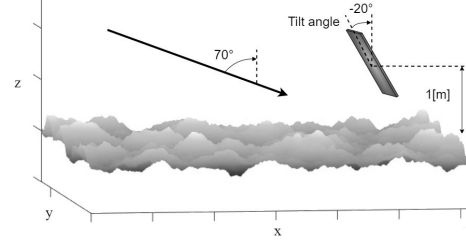


Fig. 2. Suspended PEC plate example problem.

Consider the square PEC plate shown in Figure 2, tilted -20° from vertical, with side-lengths 0.5 meters, suspended 1 meter above a rough surface, and whose scattering is solved using the physical optics approximation. The rough surface is statistically represented with the Pierson-Moskowitz spectrum as a function of windspeed above the surface, U , with relative permittivity of $\epsilon_r = 40 - 40j$. The wave illuminating this environment has an incidence angle of 70° and a frequency sweep from 8-12 GHz was used to in the computation of the FFT. The resulting range profiles numerically calculated using this analytical method are shown in Fig. 3 for varying windspeeds. Note that for the flat surface case ($U = 0$ [m/s]), the multipath reflections predicted from the four-path model appear for larger range values due to the increase in path length. As surface roughness becomes larger (e.g. $U = 1.5$ [m/s]), the response of these paths are attenuated, and completely so for $U = 2.5$ [m/s], by the reduced coherent reflection coefficient. The only coherent time domain response that remains in the high roughness limit is the backscatter response of the target.

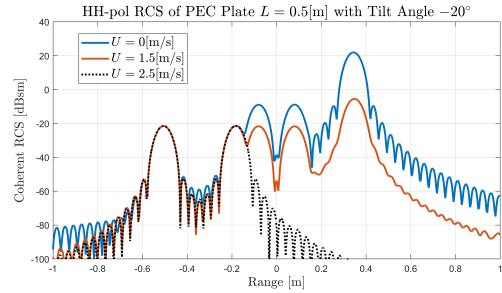


Fig. 3. Suspended PEC plate example problem coherent RCS range profile.

The fluctuating, or incoherent, field has also been derived for the decoupled method and validated with conventional methods for several target geometries and surface roughness. These results will be shown in the conference presentation.

REFERENCES

- [1] J. J. Gedney, J. T. Johnson and R. J. Burkholder, "An Analytical Formulation for the Coherent Scattered Field of a Target Above a Randomly Rough Surface," in IEEE Transactions on Antennas and Propagation, doi: 10.1109/TAP.2024.3505418.
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- [3] L. Tsang, *Scattering of Electromagnetic Waves*, vol. 3. New York: Wiley, 2001.