

Investigation of the Spectral Properties of Generalized Equivalence Integral Equations

Suryakumar Sivakumar⁽¹⁾, Yaniv Brick⁽²⁾, and Simon B. Adrian⁽¹⁾

⁽¹⁾ Universität Rostock, Rostock, Germany (suryakumar.sivakumar@uni-rostock.de;
simon.adrian@uni-rostock.de)

⁽²⁾ Ben-Gurion University of the Negev, Beersheba, Israel (ybrick@bgu.ac.il)

Method of moments integral equation solvers for electrically large problems require the use of fast methods to avoid an $\mathcal{O}(N^2)$ computational complexity (N being the number of problem unknowns, which is assumed here to scale with the frequency, for a fixed mesh density). To this end, it is convenient and common to use algebraic compression that exploits the ε -rank deficiency (ε being the best low-rank approximation error bound) of the off-diagonal moment matrix block, such as the adaptive cross approximation (ACA). If the blocks, which are organized to represent interactions between well-separated source and testing sub-domains of the scatterer Ω , exhibit ranks that scale slow with the increase in number of domain basis functions, the complexity can reach $\mathcal{O}(N \log N)$. However, for electrically large problems, this is guaranteed only for reduced dimensionality (i.e., elongated, quasi-planar) geometries, whereas for broadside interactions the ε -rank scales, asymptotically, linearly with $\mathcal{O}(N)$. Fortunately, for an essentially-convex Ω , this limitation can be overcome by formulating the problem as a generalized integral equation, with a modified kernel that includes an auxiliary component that significantly reduces the broadside interactions and highlight reduced-dimensionality interactions, thus enabling the design of fast solvers (A. Sharshevsky, Y. Brick, and A. Boag, “Direct solution of scattering problems using generalized source integral equations,” *IEEE Transactions on Antennas and Propagation*, vol. 68, no. 7, pp. 5512–5523, Jul. 2020. DOI: 10.1109/TAP.2020.2975549). The auxiliary components are produced by sources placed inside Ω , which can be designed in various manners. Their design may have influence on the spectral properties of the generalized integral equation and the conditioning of the corresponding moment matrix that, to date, have not been studied.

In this talk, we will present first results of a rigorous spectral property investigation of generalized integral equations, in the context of the various illnesses that commonly plague surface integral equations (D. R. Wilton and A. W. Glisson, “On improving the stability of the electric field integral equation at low frequencies,” in *USNC/URSI Spring Meeting Digest*, 1981, p. 24, S. B. Adrian, A. Dély, D. Consoli, A. Merlini, and F. P. Andriulli, “Electromagnetic integral equations: Insights in conditioning and preconditioning,” *IEEE Open Journal of Antennas and Propagation*, vol. 2, pp. 1143–1174, Oct. 2021. DOI: 10.1109/OJAP.2021.3121097). For convenience, the 2-D general equivalence integral equation (GEIE) (A. Boag and V. Lomakin, “Generalized equivalence integral equations,” *IEEE Antennas and Wireless Propagation Letters*, vol. 11, pp. 1568–1571, 2012. DOI: 10.1109/LAWP.2012.2236294) is used. This formulation is modified from its conventional EFIE counterpart by the introduction of a lossy obstacle inside Ω . The Green’s function (and right-hand side) should be modified to account for its scattered contribution, which effectively forces the interactions to be along the cylinder rather than through Ω . For circular and concentric Ω and internal obstacle, this formulation allows for the analytical spectral analysis, by means of the polar eigenfunctions of the modified problem defined by the two circular boundaries. The study will focus on the influence of GEIE parameters on its susceptibility/resilience to internal resonance frequencies as well as dense-discretization and high-frequency breakdowns. It will also explore the effectiveness of Calderón-type preconditioning schemes for mitigation of the latter.

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