

Recent Improvements in Accurate and Efficient Numerical Evaluation of Vertex Functions for Source Integrals in the Method of Moments

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The calculation of various surface potential integrals and their derivatives for constant and linear source densities on polygonal surfaces has a long history dating back to at least the mid-20th century, with well over a hundred papers appearing on the topic. The problem has drawn renewed attention in recent years because numerical evaluation by traditional approaches for the near-field interaction integrals have emerged as a bottleneck for fast algorithms in the method of moments since they are not only more difficult to compute, but also their associated element interactions cannot be aggregated. A recent paper (R. Chang, Z. Wang, and Q. Xie, “Fast Convergent Quadrature Method for Evaluating the RWG- and SWG-Related Convolutional Integrals,” *IEEE Trans. Ant. Propagat.*, 69, 12, 2021) draws heavily on earlier work using Gauss theorems to analytically evaluate inner, radial integrals from the projection $\mathbf{r}_0$ of the observation point $\mathbf{r}$ onto the plane of a source triangle, to the opposite element boundary (edge), thus reducing the evaluations to outer line integral sums over the three source triangle edges. The integrals are accelerated using $\sinh^{-1}$, double- $\sinh^{-1}$, and $\tan^{-1}$ transforms on an ad hoc basis.

Recently, the authors have reported several important improvements in understanding, representing, applying, and enhancing convergence of these integrals. They include the following: i) Splitting of the usual three-adjacent-subtriangle subdivision, into six *right*-subtriangles with $\mathbf{r}_0$, the projection of observation point $\mathbf{r}$ onto the triangle plane, as their common vertex, while the remaining vertices are those of the original source triangle. The three heights h (from $\mathbf{r}_0$) of the original sub-triangles each becomes a common side for two of the right-subtriangles, with remaining side-lengths denoted by l ; the distance d measured from $\mathbf{r}_0$ to $\mathbf{r}$ is common to all subtriangles. The so-called *vertex potential* $I(h,l,d)$ for each right-subtriangle embodies the subtriangle geometry, potential kernel, and source at vertex or edge (h,l,d) = (0,0,0) or (h,d) = (0,0), respectively. ii) Simplification of the resulting integrals and their behavior at $\mathbf{r}_0$ by explicitly integrating radially from $\mathbf{r}_0$ to the opposite edge, ensuring both continuity across adjacent subtriangle edges and vanishing of the integrands at $\mathbf{r}_0$. iii) Regularization of all integrands such that limiting cases $k \rightarrow 0$ and $R \rightarrow 0$ exist, simultaneously factorizing each into a product of static and dynamic factors, with the latter approaching unity as $k \rightarrow 0$. iv) Acceleration of the transformed integral via a transform *exactly* canceling the static integrand factor, leaving only a smooth dynamic factor (≈ 1 , $|kl| \ll 1$) to be integrated by Gauss-Legendre quadrature along right-subtriangle edges.

In this work, we briefly review the above points, then extend the representation showing that the original four integrals required to represent scalar potential, its gradient, and vector potential can be reduced to the evaluation of four vertex function integral types, each requiring either a $\sinh^{-1}$ or $\tan^{-1}$ transform, both with explicit inverses, hence eliminating the need for root-finding to invert the accelerating transform. The resulting integral evaluation is robust, accurate, and highly convergent. Finally, we will investigate the use of a trained machine learning algorithm to determine the minimum number of integration sample points and weights required for each potential line integral to obtain the requested accuracy.