

Efficient well-conditioned discretization of the MFIE

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Perfect electric conductors (PECs) and dielectric objects can be efficiently modeled by boundary integral equations (BIEs). The electric and magnetic field integral equations (EFIE and MFIE) are used to model scattering by PEC geometries. Accurate discretization of the EFIE and MFIE requires a primal finite element space (e.g. Rao Wilton Glisson, RWG space) and a dual one on the mesh refinement (e.g. Buffa-Christiansen, BC space). The EFIE requires the dual discretization for preconditioning, while the MFIE requires it to get a non-singular well-conditioned matrix. The MFIE could be discretized without BC functions, yet the choice of the test functions is not obvious: if tested with RWG functions, the MFIE leads to a well-conditioned system but the solution is affected by larger errors compared to the EFIE, while if tested with rotated RWG functions the system results ill-conditioned. Testing the MFIE with rotated BC functions, instead, produces a well-conditioned system *and* accurate results. Nevertheless, the discretization on the refinement has the drawback of being computationally expensive. A stable refinement-free discretization is still a desirable feature.

In (Christiansen et al, SIAM J. Numer. Anal. 40 (2002), no. 3, 1100-1135), a refinement-free approach was proposed to precondition the EFIE. Based on a Helmholtz decomposition of the primal space, this approach builds a map Θ approximating the inverse of the $\hat{n} \times$ operator. In (A. Zuccotti, K. Cools, AP-S/INC-USNC-URSI, Firenze, Italy, 2024, pp. 1327-1328) the Θ map is used to discretize products of both EFIE and MFIE operators leading to a refinement-free discretization of the single-source equation. Here, we propose to implement a well-conditioned discretization of the MFIE by testing it with a set of rotated RWG functions and multiplying the system with the transpose of Θ . This procedure numerically builds a subspace on which the MFIE is well-tested. It was proved indeed that the image of $\Theta^\dagger$ consists of a subspace of RWG space on which a well-approximating solution can be efficiently constructed by iterative solvers. This formulation has the benefit of reducing the implementation task to only the RWG space. We highlight that the definition of Θ involves the inversion of a saddle point system for which an efficient iterative solution is possible. The computational cost of assembling such a system is significantly reduced with respect to the discretization on the refinement. Numerical experiments have been run by following this discretization, for a PEC sphere and a cuboid: in both cases, the system requires a greatly reduced number of iterations when multiplied by $\Theta^\dagger$. However, the matrix system is singular, and a slight increase in the iterations was observed when reducing the mesh size. Errors have been estimated by comparing the solutions with the solution on a much finer mesh given by the well-established discretization with RWG and BC functions. Such errors result in the same order as those given by the discretization on the refinement, with an analogous scaling with the mesh size. The obtained MFIE presents good efficiency, simple implementation, and comparable precision with respect to the discretization on the refinement. Such formulation is also amenable to higher-order discretization. Following the work on preconditioning for higher order discretizations of the EFIE (J. Bourhis et al, ICEAA, Lisbon, Portugal, 2024, pp. 700-703), we conducted a study of the spectral properties of our novel MFIE discretization. In accordance with the findings of Bourhis et al., we discovered that choosing a mixed discretization with a primal space of RWG lowest order $p = 0$ and an auxiliary space of order $p = 2$ leads to a non-singular well-conditioned system. In this case, the number of iterations needed to solve the system is independent of the mesh size.

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