

Study of a B-Spline Based Calderón Preconditioner for the Electric Field Integral Equation

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The electric field integral equation (EFIE) is a widely employed method for solving electromagnetic scattering and radiation problems. However, it is susceptible to two significant challenges: the low-frequency breakdown and the dense-discretization breakdown, which lead to poor or no convergence of iterative solvers.

An effective approach for overcoming these breakdowns is the use of preconditioners derived from Calderón identities. By leveraging the self-regularizing property of the EFIE, these preconditioners exploit the fact that the EFIE operator, when applied to itself, yields a second-kind integral operator. This resulting operator comprises an identity operator augmented by a compact perturbation.

However, preconditioning the EFIE using a Calderón preconditioner poses significant challenges. Specifically, the two EFIE operators must be linked through the inverse of the Gram matrix, which establishes a connection between the expansion functions of the second EFIE operator and the testing functions of the first EFIE operator. In the context of Rao-Wilton-Glisson (RWG) based discretizations, this has been achieved by incorporating dual basis functions, namely the Buffa-Christiansen (BC) functions. As a consequence, a purely multiplicative preconditioner has been established (F. P. Andriulli, K. Cools, H. Bagci, F. Olyslager, A. Buffa, S. Christiansen, and E. Michielssen, “A multiplicative Calderón preconditioner for the electric field integral equation,” *IEEE Trans. Antennas Propag.*, vol. 56, no. 8, pp. 2398–2412, Aug. 2008).

Recently, we proposed an approach to establish such a multiplicative Calderón preconditioner also for a B-spline-based discretization of the EFIE (with arbitrary polynomial degree) on descriptions of the surface by non-uniform rational B-splines (NURBS) patches (B. Hofmann, T. F. Eibert, F. P. Andriulli, and S. B. Adrian, “A multiplicative Calderón preconditioner for the B-spline-based electric field integral equation,” in *Int. Conf. Electromagn. Adv. Appl. (ICEAA)*, Sep. 2024, pp. 447–450). A key benefit of this (higher-order) discretization strategy is that it circumvents the necessity of generating a mesh, and it fully avoids the introduction of geometrical approximation errors.

This work presents further investigations, analyses, and results regarding the B-spline-based multiplicative Calderón preconditioner and the corresponding set of higher-order dual basis functions that we have proposed. Notably, the latter can be expressed explicitly as a superposition of basis functions on a refined parametric space. In the lowest order case, they correspond precisely to BC functions on (curvilinear) rectangular patches and can, hence, be considered a generalization of the BC functions to arbitrary polynomial degrees.

This work was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – 504345461.