

Analytical Methods and Design Approaches for Time-modulated Metastructures

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Abstract— Time-varying metamaterials are artificial electromagnetic material whose properties dynamically change over time. Among the different modulation waveforms, the step-like modulation has attracted the interest of the scientific community because of the possibility to realize the temporal analog of a spatial interface. Recently, we studied the scattering process taking place in a time-varying metastructures characterized by a single, double, and multiple interfaces over time in both infinitely extended media and in guiding structures. In this contribution, we specifically focus on the analytical methods that have been used for modeling and designing them, with particular emphasis on the temporal transfer matrix, mode-matching and Laplace method.

I. INTRODUCTION

The unprecedented control of wave propagation provided by metamaterials and metasurfaces has been recently extended thanks to the time-modulation of their properties [1]–[3]. This unlocked new capabilities, such as tailoring the temporal characteristics of the electromagnetic (EM) waves and overcoming fundamental bounds constrained by passivity. A list of relevant examples includes the possibility to realize matching conditions that overcome the Bode-Fano limit [4], temporal Fabry-Perot slab resonators [5], wave aiming [6], effective temporal media [7], absorbers with extended bandwidth [8], lasing media [9], temporal metasurfaces [10]–[11], and efficient phase conjugation [12]. All these works require an abrupt change of material properties over time in the entire propagation domain, making it complex to be realized in realistic scenarios. Some attempts to relax this constraint have been proposed, considering the propagation along a time-switched transmission line [13] or in optical waveguides [14]–[16]. Another approach was based on the effective medium properties perceived by guided waves. By controlling the boundary conditions of guiding systems, it is possible to induce temporal interfaces where a similar scattering process takes place. Recently, three anomalous scattering processes have been investigated: *i*) the generation a set of forward and backward propagating modes, each of which perceive a different change of the material properties [17], *ii*) the diffusion of a continuous spatio-temporal spectrum of plane waves [18], *iii*) the radiation of a directive beam in end-fire direction whose frequency is much higher than the source frequency [19].

This contribution focuses on the analytical methods and design approaches that have been used for modeling and designing the time-modulated metastructures with abrupt changes of the bulk or effective material properties. We briefly introduce the analytical method and how it has been used for

the specific structure at hand. In Fig. 1 we report a summary of the time-modulated metastructures that will be presented, classified according to the corresponding analytical method used to derive the characteristics of the scattered waves from the time interface.

II. ANALYTICAL METHODS FOR TIME METASTRUCTURES

A. Temporal transfer matrix method

The transfer matrix method (TTm) is a powerful tool for analyzing the propagation and scattering of EM waves through stratified mediums. Recently, by exploiting the concept of space-time duality, the temporal counterpart of the TTm has been developed enabling the analysis and design of temporal metastructures consisting of a cascade of temporal interfaces.

As illustrated in Fig. 1(a)–(d), the temporal TMm has been successfully used for analyzing the reflection and transmission from a temporal slab [5] and a multilayered structure [9], where the propagation matrix has been replaced by the time-delay matrix. This allows to derive the scattering response in terms of the new parameter *effective application time*, that plays the same role of the electric thickness for static spatial EM structures. More details can be found in [5], [9].

B. Mode-matching method

The mode-matching method (MMm) is a very useful technique for formulation of boundary-value problems. It is based on matching the fields at the spatial boundaries of different regions, making it perfectly suitable for the analysis of boundary-value problems. MMm has widely been used for scattering and transmission problems, as well as transmission line analysis, especially in case of discontinuities in waveguides.

As illustrated in Fig. 1(e)–(i), this method has been applied to derive the amplitude of the scattered waves at a boundary-induced time interface. We successfully analyze the scattering process taking place within a parallel-plate waveguide in two different scenarios: the first one considers the case of sudden variation of the distance between the two plates as in [17], whereas the second one considers the case of sudden disappearance of the top plate as in [18]. The method allows deriving the amplitude of the scattered modes excited in the new waveguide for different distances of the plates (Fig. 1(f)–(g)) and the energy distribution of the temporal rainbow emerging from the time-switched metastructure (Fig. 1(i)). More details on this analytical approach can be found in [17], [18].

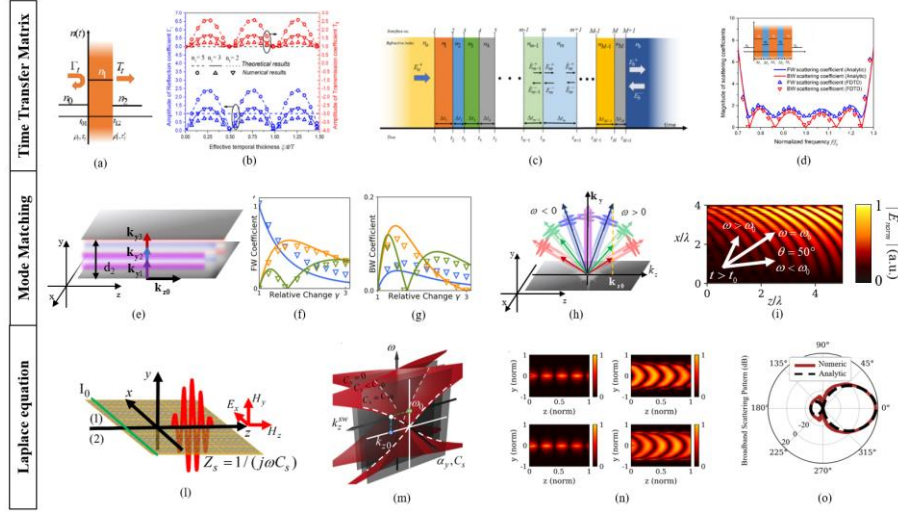


Figure 1. (a)-(d) Temporal multilayered structures and corresponding results obtained by using the temporal transfer matrix method; (e)-(i) Parallel-plate waveguide platforms for achieving boundary-induced time interfaces and corresponding results obtained by using mode matching method; (l)-(o) Time-switched metamaterial and corresponding results obtained by solving the wave equation in the Laplace domain with proper initial values.

C. Laplace method

The Laplace method (Lm) exploits the Laplace transform of wave equation to derive compact and physically transparent solution for the EM field emerging from time-modulated metastructures. By using Laplace transform, one can easily accommodate the initial value problems and derive the formally exact solution in the form of an inverse Laplace transform integral.

As illustrated in Fig. 1(i)-(o), such a method has been employed to derive the scattered field from the metasurface and the corresponding radiation pattern. The method allows connecting the original surface wave before the time switching, that is confined along the metasurface due to the finite surface impedance, with the radiated set of planewaves emerging from the metasurface after the switching time. More details on this analytical approach can be found in [19].

III. CONCLUSIONS

In this contribution, we have presented the analytical methods that have been used for modeling and designing time-modulated metastructures, with particular emphasis on the temporal transfer matrix method, mode-matching method and Laplace method.

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