

Extended GSTCs and Multipolar Modeling of Spatially Dispersive Metasurfaces

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Metasurfaces are 2D arrays of resonating particles whose microscopic properties can be tailored to produce a desired macroscopic field scattering response. A particularly interesting class of metasurfaces is nonlocal or spatially dispersive (SD) metasurfaces. For these surfaces, the induced currents on the surface are related to fields over an extended region of the structure. This nonlocality allows for the control of the angular scattering of the metasurface. The potential to tightly control the angular scattering response of a surface has led to SD metasurfaces finding potential applications in wave-based computing and space compression.

In the literature, two techniques have emerged as a means to model these surfaces. Both techniques are an extension of the Generalized Sheet Transition Conditions (GSTCs) technique where the surface is characterized by a zero-thickness sheet with dipolar surface susceptibility tensors and where the fields around the surface must satisfy the GSTCs. The limitation of this model lies in the fact that the induced currents are locally related to the fields at the surface.

The first technique to model SD surfaces is the multipolar GSTCs technique. In this this method, higher-order multipoles are considered beyond the standard dipolar treatment. Hypersusceptibilities are then used to relate the induced multipolar moments to the fields around the surface and their derivatives. The derivatives of the fields at the surface capture the inherent nonlocality of the metasurface.

The second technique is known as the Extended GSTCs. In this method the dipolar susceptibilities are functions of the transverse wavevector. In the spatial domain, this is equivalent to relating the induced currents to a convolution of the fields over the surface capturing any nonlocal effects. If the extracted susceptibilities takes the form of a rational polynomial in the spatial frequency domain, then we can conveniently represent the SD metasurface as a higher-order boundary condition in the spatial domain. This method has been implemented in a variety of numerical solvers such as an Integral Equation solvers, FDTD, and plane wave expansion. Further, it has been shown to give accurate results for both uniform and non-uniform surfaces.

In a previous work, the equivalence between these two methods was explored on a theoretical basis. It was demonstrated that the multipolar GSTC method is equivalent to the Extended GSTC method if the dipolar GSTCs take the form of polynomials in the spatial frequency domain. From this, we can draw the conclusion that the Extended GSTCs are more flexible as the rational polynomial form allows for the modeling of resonances in the angular domain, which have been shown to exist in practical metasurface structures. However, the the multipolar GSTCs, are more grounded in the physics of the unit cell. Therefore, further exploration on the relationship between these two methods is needed in so that the flexibility of the Extended GSTCs can be combined with the physical rigor of the multipolar approach.

As a first step in this comparison we plan to perform an extraction of a physical unit cell using both of these methods. This extraction will highlight the trade offs between the two modeling approaches while providing valuable insight into the relationship between the angle dependent dipolar susceptibilities and the multipoles present in the unit cell. We believe this work will be critical in developing a physics based understanding of the rapidly growing field of nonlocal metasurfaces. This physics based understanding will be crucial in developing techniques for the physical realization of SD responses.