

Exploring Aperiodic Order in Photonic Time Crystals

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Photonic time crystals (PTCs) have emerged as one of the most fascinating developments in the study of time-varying media (M. M. Asgari *et al.* “Theory and Applications of Photonic Time Crystals,” *Adv. Opt. Photon.*, vol. 16, pp. 958-1063, 2024). These structures can be viewed as the temporal analog of photonic crystals, featuring *temporal* modulations of their constitutive parameters, with time scales comparable to the electromagnetic wave period. Like their spatial counterparts, PTCs exhibit strong dispersion; however, their band structures are characterized by *momentum* (wavevector) bandgaps instead of *frequency* bandgaps. Remarkably, due to the non-conservation of energy in these systems, momentum bandgaps are associated with exponential signal amplification, as energy is extracted from the temporal modulation. These unique properties open up intriguing possibilities in both fundamental research and practical applications, including thresholdless lasers, Cherenkov radiation, optical absorbers, and enhanced resolution imaging.

The majority of the literature on PTCs has concentrated on *periodic* temporal modulations. However, a few notable exceptions have investigated the effects of *random disorder* (Y. Sharabi *et al.*, “Disordered Photonic Time Crystals,” *Phys. Rev. Lett.*, vol. 126, 163902, 2021) and *almost-periodicity*, characterized by the superposition of noncommensurable harmonics (S. Fr. Koufidis *et al.*, “Enhanced Scattering from an Almost-Periodic Optical Temporal Slab,” *Phys. Rev. A*, vol. 110, no. 4, L041501, 2024). These studies suggest that relaxing the assumption of strict periodicity can offer significant advantages, unveiling new phenomena and enhancing performance.

In our ongoing research, we have developed a rigorous theoretical framework for modeling *aperiodically ordered* PTCs using general two-symbol substitution sequences, leveraging the trace-map and anti-trace map formalism (X. Wang *et al.*, “Trace and Antitrace Maps for Aperiodic Sequences: Extensions and Applications,” *Phys. Rev. B*, vol. 62, no. 21, pp. 14020–14031, 2000). This approach facilitates the investigation of temporal analogs to well-known spatial photonic “quasicrystals,” such as those based on Fibonacci or Thue-Morse sequences. Similar to their spatial counterparts, these “photonic time quasicrystals” exhibit complex band structures and band-edge states, possibly characterized by fractal scaling, and potentially offer greater robustness to imperfections. In our presentation, we will explore these analogies, highlight the fundamental differences, and discuss their implications and potential applications.