

An Adaptive Local Time-stepping Discontinuous Galerkin Method for Resistive Boundary Condition with Field and Time-dependent Conductivity

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In recent years, two-dimensional (2D) materials, such as graphene, have gained significant importance in electromagnetic systems. To optimize simulations of these systems and reduce the computational cost associated with modeling the thin layers of 2D materials using fine meshes, an infinitesimally thin sheet with a resistive boundary condition (RBC) is often employed as a substitute for the thin layer. Depending on the material being replaced by the effective representation, the conductivity of the RBC might vary. In some cases, it might be a nonlinear function of the electric field (C. Valagiannopoulos, A. Sarsen, and A. Alù, *IEEE Trans. Antennas Propag.*, vol. 69, pp. 7720–7728, 2021). This property enables many intriguing applications, including hologram creation and all-optical control. Additionally, the conductivity can be time-varying (K. Schab, B. Shirley, K. C. Kerby-Patel, *IEEE Trans. Antennas Propag.*, vol. 70, pp. 7011–7023, 2022). Such time-varying behavior can be achieved in metasurfaces (treated as 2D materials) through dense networks of lumped time-varying components or by modulating inherently surface-based materials to enable frequency conversion.

Problems involving RBCs with nonlinear and time-varying conductivity are best simulated by time-domain electromagnetic solvers. Among these methods, the finite difference time-domain (FDTD) and finite element time-domain (FETD) approaches are prominent. While FDTD is often computationally efficient, it suffers from stair-casing errors. In contrast, FETD allows for more accurate representation of arbitrarily shaped geometries but requires solving a large matrix equation at each time step, significantly increasing computational costs. Recently, the discontinuous Galerkin time-domain (DGTD) method (J. S. Hesthaven and T. Warburton, New York, NY, USA: Springer, 2007) has gained considerable interest for addressing electromagnetic problems. As a domain decomposition method, DGTD treats each element as a subdomain connected to its neighbors via numerical flux, applying the finite element method to each element. Consequently, DGTD involves solving a block diagonal matrix equation at each time step, reducing the computational overhead associated with traditional FETD.

RBCs are incorporated into DGTD by modifying the numerical flux (P. Li, L. J. Jiang and H. Bagci, *IEEE Trans. Antennas Propag.*, vol. 63, pp. 3065–3076, 2015). However, directly applying the explicit Runge-Kutta method for time integration in DGTD for a problem involving a RBC can cause instability when the conductivity value is high. This instability arises because of the larger eigenvalues of the system matrix associated with the large conductivity. In this work, an adaptive local time-stepping DGTD scheme is proposed for efficient simulation of problems involving RBCs. A small time-step size is assigned to the elements “touching” the RBC, while a large time-step size is used for the other elements. To further increase the efficiency for cases when the RBC’s conductivity is time-dependent or nonlinear, the time-step size assigned to the RBC elements is allowed to vary dynamically during time marching to ensure that the largest eigenvalue of the system matrix remains below 1.0. The information exchange between different elements with different time-step sizes is done via interpolation.

Numerical examples, which demonstrate the accuracy, efficiency, and applicability of the proposed method, will be presented at the conference.