

Analytical derivation of a resistive sheet model for optimization or sensitivity analysis purposes

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In this paper, we compute and implement an analytical derivation of a type of impedance condition for a Boundary Element Method and illustrate how such feature can be used in an optimization process. The goal would be to optimize the shape of the radiating PEC patch of an antenna using topology optimization techniques. Such techniques are widely used in mechanical engineering and aim at optimizing material density functions. In our approach, the local conductivity of the patch would play the role of the density of the material: a high conductivity would mean presence of the PEC while a low conductivity would mean absence of the PEC. We model a thin layer of imperfectly electric conductor using the classical impedance per square model that links the tangential electric fields and magnetic fields on each side of the layer through the impedance Z as

$$\frac{1}{2} (E_{tg}^- + E_{tg}^+) = Z (n \times H^+ - n \times H^-), \quad Z = \frac{1}{\sigma d}, \quad (1)$$

where σ is the conductivity of the conductor and d its thickness. In the EFIE framework of the Boundary Element Method, this corresponds to adding a single integral term $\int_{\Sigma} Z(x) j(x) \cdot j'(x) dx$ to the classical weak formulation, where j' are the classical RWG basis functions.

In matrix form, the system writes $(\mathbb{A} + \mathbb{M}(Z)) J(Z) = J_{inc}$, and derivating it with respect to the local impedance z_k of the element k , one can show that

$$\frac{\partial J}{\partial z_k} = -(\mathbb{A} + \mathbb{M}(Z))^{-1} \frac{\partial \mathbb{M}}{\partial z_k} (\mathbb{A} + \mathbb{M}(Z))^{-1} J_{inc} = -(\mathbb{A} + \mathbb{M}(Z))^{-1} \frac{\partial \mathbb{M}}{\partial z_k} J(Z). \quad (2)$$

From this expression, it is easy to compute the derivative of several other quantities of interest with respect to z_k , including the scattered far field, the gain of an antenna, its radiation pattern or its S parameters, etc. In practice, the computation of these derivatives with respect to the local impedance of the N elements of the surface require one first solve to obtain $J(Z)$ the solution of the initial system, and then N solves of the same system with specific right-hand sides. Note that the computation of the products $\frac{\partial \mathbb{M}}{\partial z_k} J(Z)$ to form the right-hand sides is very cheap since the matrices involved are very sparse (9 non-zero elements per matrix).

We have implemented this formulation and its derivation in the Arlene code, developed at CEA. This is a highly parallel MPI+OpenMPI code, implementing a wide range of BEM formulations with an "in-house" dense and $\mathcal{H}$ -matrix direct solver. Considering the number of resolutions required, the use of a direct solver is a clear advantage here.

During this talk, we will present the derivation of the formulation, its implementation and validation results. We will also present a first few optimization results on simple cases.