

Performance of Implicit Local Error Estimators for Integral Equation Based Numerical Methods

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The performance of several local error estimators for perfectly conducting (PEC) targets treated with the electric field integral equation (EFIE) and the Rao-Wilton-Glisson (RWG) basis functions has been reported (Kim and Peterson, "Evaluation of local error estimators for the RWG based EFIE," *IEEE Trans. Antennas Propagat.*, vol. 66, pp. 819-826, 2018; Steines, Baxley, and Peterson, "Performance of inexpensive local error estimation techniques for integral equation numerical solutions," *IEEE J. Multiscale and Multiphysics Comp. Tech.*, vol. 8, pp. 158-164, 2023). These estimators are an essential part of an adaptive refinement algorithm for optimizing the performance of numerical solution schemes. Adaptive refinement involves adjusting the cell sizes or representation order in sub-regions of the target where the current density exhibits excessive error, to achieve necessary accuracy in a more efficient way than by uniform mesh refinement. For integral equation formulations, error estimation can be relatively expensive and has not been widely adopted. This has hindered the development of adaptive refinement schemes for integral equation problems.

Inexpensive error estimators are based on surrogates such as local discontinuities in the current density or charge density, recovery algorithms that construct a smoother version of the current density for comparison, or "implicit" approaches that successively refine small parts of the mesh to investigate the solution's sensitivity to change in that region. Implicit refinement involves first obtaining a numerical solution on a relatively coarse mesh using a method of moments (MoM) approach. Then, a series of smaller problems involving regions of cells surrounding each target cell are solved using either a finer mesh (h -implicit) or higher order representation (p -implicit) to assess the extent to which the results change. The key idea is to re-use the results of the initial MoM solution outside of the region under consideration. In principle, this approach enables an efficient estimation scheme involving many small problems.

A previous h -implicit approach was favorably reported in connection with an RWG solution of the EFIE (Kim and Peterson, 2018). In this presentation, the performance of implicit local error estimators based on both local h -refinement and local p -refinement schemes will be investigated. Questions of interest center around the optimum size of the footprint of cells surrounding a target cell, the accuracy of the estimated error, and the computational efficiency of the approach. A variety of targets including spheres, cone-spheres, NASA almonds, simple missiles, and other aircraft shapes illuminated by plane waves are considered. Reference results are obtained from solutions obtained with uniformly higher mesh densities or uniformly higher degree basis functions. Correlation coefficients will be used to assess the relative accuracy of these estimators. The numerical efficiency of the implicit schemes will also be evaluated. The overall performance of the implicit estimators will be compared with other inexpensive estimators including current discontinuity and current recovery approaches, and more expensive estimators involving residual calculations.