

Investigating a B-Spline Based Acceleration Method for the Electric Field Integral Equation

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Solving radiation and scattering problems, particularly those involving perfectly electrically conducting objects, by leveraging the electric field integral equation (EFIE) in conjunction with the boundary element method, is a widely used approach. However, a significant drawback of this method is the resulting densely populated system matrix, which restricts its application to problems with a small number of unknowns due to computational and storage constraints. To address this limitation, matrix-free acceleration techniques such as the propagating plane-wave based multilevel fast multipole method (MLFMM) (W. Chew, E. Michielssen, J. M. Song, and J. M. Jin, *Fast and Efficient Algorithms in Computational Electromagnetics*. Artech House, Inc., Jul. 2001), the adaptive cross approximation (ACA) (K. Zhao, M. Vouvakis, and J.-F. Lee, “The Adaptive Cross Approximation Algorithm for Accelerated Method of Moments Computations of EMC Problems,” *IEEE Transactions on Electromagnetic Compatibility*, vol. 47, no. 4, Nov. 2005), or the interpolatory $\mathcal{H}^2$ -method (W. Hackbusch and S. Börm, “ $\mathcal{H}^2$ -matrix approximation of integral operators by interpolation,” *Applied Numerical Mathematics*, vol. 43, no. 1–2, Oct. 2002; D. T. Schober and T. F. Eibert, “Low-Frequency Surface Integral Equation Solution by Multilevel Green’s Function Interpolation With Fast Fourier Transform Acceleration,” *IEEE Transactions on Antennas and Propagation*, vol. 60, no. 3, Mar. 2012) have been developed. These techniques reduce the storage and computational complexity of a matrix-vector product from quadratic to (quasi-)linear. As a consequence, they enable the efficient solution of larger and more complex problems when combined with an iterative solver.

To achieve (quasi-)optimal complexity, the MLFMM employs a kernel dependent addition theorem together with a propagating plane-wave expansion. However, due to numerical instabilities at low frequencies, the method is best suited for mid- to high-frequency applications, where the problem size is larger than the considered wavelength. The ACA on the other hand, as a purely algebraic method, and the interpolatory $\mathcal{H}^2$ -method both operate in the low-frequency regime.

In this work, we employ the interpolatory $\mathcal{H}^2$ -method, which offers error controllability, allowing for reliable error estimation. The interpolatory $\mathcal{H}^2$ -method, as originally put forward, employs Lagrange polynomials to interpolate the Green’s function of the EFIE. This polynomial interpolation enables the establishment of error controllability. To achieve a multilevel method, first, an octree subdivides the geometry under inspection. Based on the octree, polynomials on a higher level are then interpolated exactly by their child polynomial on the subsequent level.

While still utilizing polynomials, this work explores an alternative approach, examining the potential of using B-spline interpolation in the $\mathcal{H}^2$ -method. The locality of the B-splines is leveraged for faster translations between subsequent levels in the multilevel framework in contrast to globally-supported Lagrange polynomials. Furthermore, we examine the error controllability and the computational efficiency, comparing assembly times and matrix-vector product timings to the original method.

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