

Discrete sources schemes in 2-D scattering: the method of auxiliary sources versus the null-field method

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Abstract—The numerical schemes of the method of auxiliary sources (MAS) and the null-field method (NFM) are discussed and compared with reference to two-dimensional scattering problems by dielectric scatterers. Similarities in the linear systems obtained by the two methods are pointed out. Essential differences in the convergence properties for large numbers of auxiliary sources involved are examined.

I. INTRODUCTION

The method of auxiliary sources (MAS) [1], [2] and the null-field method (NFM) [3] (or extended integral equation (EIE) method) are two established numerical methods for treating scattering and radiation problems for acoustic, electromagnetic, and elastic waves. In this work, we examine a version of the NFM incorporating the surface equivalence principle (SEP) [4], [5]; this version is referred to hereafter as NFM-SEP. Our main purpose is to point out similarities between the MAS and the NFM-SEP and emphasize fundamental differences in their convergence properties.

In case of two-dimensional (2-D) scattering problems by non-penetrable scatterers, the MAS and the NFM-SEP share some common characteristics like, e.g., the involvement of an auxiliary curve displaced from the physical boundary. In MAS, this auxiliary curve serves as the support of the auxiliary sources generating the scattered field, while in NFM the extended boundary condition is applied on a set of points lying on this auxiliary curve. As a result, the obtained linear systems by the two methods have a very close resemblance, while in fact the matrix involved in the one method is the transpose of that of the other method. Despite such striking similarities, it has been shown in [6] that, for perfect electric conducting (PEC) scatterers, the two methods exhibit some very different convergence properties regarding the behavior of the auxiliary sources' strengths and the computed fields in the large- N limit, where N is the number of auxiliary sources involved. We revisit this convergence analysis (originally developed for non-penetrable shapes) with the main purpose of presenting some recent new results on scattering by dielectric shapes. It is shown that certain convergence characteristics of NFM-SEP are in stark contrast with the corresponding ones of MAS despite that the two methods share actually the same matrix.

II. MAS FOR 2-D PROBLEMS

The application of the MAS for the problem of internal or external line-source excitation of a dielectric cylinder is

presented in [7]. In case of external excitation with primary filament I of a circular dielectric cylinder with radius ρ_{cyl} , the application of MAS is depicted in Fig. 1.

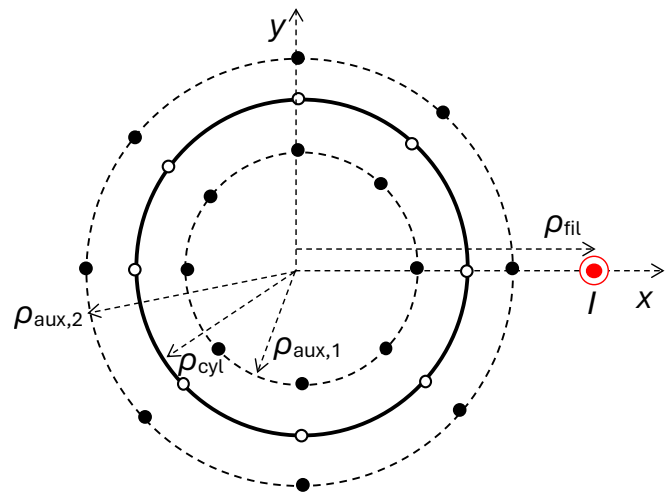


Fig. 1. Application of the MAS and the NFM-SEP for excitation of a circular cylinder by an external line source. The meaning of the black and white bullets depends on which method is considered and is explained in the text. Both methods involve two auxiliary surfaces with radii $\rho_{\text{aux},1} < \rho_{\text{cyl}}$ and $\rho_{\text{aux},2} > \rho_{\text{cyl}}$, respectively.

It involves two auxiliary surfaces with radii $\rho_{\text{aux},1}$ and $\rho_{\text{aux},2}$ each hosting a number N of auxiliary sources (ASs); these are shown as black bullets in Fig. 1. The ASs on $\rho_{\text{aux},1}$ simulate the scattered field (for $\rho > \rho_{\text{cyl}}$), while the ones on $\rho_{\text{aux},2}$ simulate the internal field (for $\rho < \rho_{\text{cyl}}$). The transmission boundary conditions are, subsequently, applied at N collocation points (CPs) on the actual (physical) boundary $\rho = \rho_{\text{cyl}}$; these are shown as white bullets in Fig. 1. In this way, a linear system is formulated with respect to the unknown ASs currents on $\rho_{\text{aux},1}$ and $\rho_{\text{aux},2}$.

In [7], it was shown that when

$$\rho_{\text{aux},1} < \frac{\rho_{\text{cyl}}^2}{\rho_{\text{fil}}} \quad \text{or} \quad \rho_{\text{aux},2} > \rho_{\text{fil}}, \quad (1)$$

and for $N \rightarrow \infty$, the corresponding ASs currents diverge and oscillate, yet, the computed MAS fields generated by these currents converge to the respective exact fields.

III. NFM-SEP FOR 2-D PROBLEMS

According to the SEP, surface equivalent current densities are defined on each boundary of a scattering problem [8], [9]. Imposing the boundary conditions leads to a system of surface integral equations of equal number to that of the unknown densities. Typically, the method of moments is used to solve these equations, which however encounters difficulties with the singular integrals in the diagonal elements of the matrix. The singularities in the integrals are due to that the integration domain coincides with the physical boundary. Hence, alternative methods were developed in which the support of the current distribution is displaced from the domain on which the boundary conditions are applied [10], [11].

The scattering problem considered is the same with that of Section II. Now, we apply the SEP [4], [5], according to which two equivalent problems of the original are defined, the first for the fields in the external and the second for those in the internal region. For the external problem, two surface current densities, an electric $\mathbf{J}^s$ and a magnetic $\mathbf{M}^s$, are defined on the physical boundary C (i.e., $\rho = \rho_{\text{cyl}}$), as follows:

$$\mathbf{J}^s = \hat{\mathbf{n}} \times \mathbf{H}, \quad \mathbf{M}^s = \mathbf{E} \times \hat{\mathbf{n}}, \quad (2)$$

where $\hat{\mathbf{n}}$ is the outward normal unit vector on C , while $\mathbf{E}$ and $\mathbf{H}$ is the total electric (magnetic) field just outside C . For the second equivalent (internal) problem, we need to define the following corresponding surface current densities $\tilde{\mathbf{J}}^s$ and $\tilde{\mathbf{M}}^s$ on C

$$\tilde{\mathbf{J}}^s = -\mathbf{J}^s, \quad \tilde{\mathbf{M}}^s = -\mathbf{M}^s, \quad (3)$$

i.e., the opposites of those defined in the first equivalent problem.

To implement the SEP, it is first required to calculate $\mathbf{J}^s$ and $\mathbf{M}^s$. This is accomplished by means of a system of two equations resulting from two boundary conditions. In the context of NFM, the first boundary condition arises from the first equivalent problem and requires the zeroing of the total (electric or magnetic) field on an auxiliary surface $\rho = \rho_{\text{aux},1}$ inside C . The second boundary condition stems from the second equivalent problem and requires the zeroing of the total field on an auxiliary surface $\rho = \rho_{\text{aux},2}$ outside C . In this way, a system of linear integral equations is obtained.

Now, we turn to the discretized version of the NFM-SEP the scheme of which is depicted in Fig. 1. The continuous electric and magnetic current densities are approximated by a set of N discrete sources; these are shown as white bullets in Fig. 1. Specifically, the electric current density $\mathbf{J}^s$ is replaced by N infinitely long z -directed electric current filaments with currents I_l . The magnetic current density $\mathbf{M}^s$ is replaced by N magnetic current filaments, each carrying a voltage K_l , with $l = 0, 1, \dots, N-1$. Then, to determine the electric I_l and magnetic K_l current amplitudes, we satisfy the boundary conditions at N CPs on $\rho = \rho_{\text{aux},1}$ and $\rho = \rho_{\text{aux},2}$; these are shown as black bullets on each of the two auxiliary surfaces in Fig. 1. This leads to a $2N \times 2N$ linear system with unknowns the complex amplitudes I_l and K_l . This system is written in

block matrix form as

$$\begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I} \\ \mathbf{K} \end{bmatrix} = \begin{bmatrix} \mathbf{V} \\ \mathbf{0} \end{bmatrix}, \quad (4)$$

where $\mathbf{I}$ and $\mathbf{K}$ are N -dimensional vectors with elements I_l and K_l , respectively, while the N -dimensional vector $\mathbf{V}$ incorporates the contributions from the excitation sources. The 2×2 block matrix of the NFM-SEP system is composed of circulant blocks or order N . Hence, the system (4) can be solved explicitly by applying techniques based on the discrete Fourier transform.

Importantly, the matrix of the NFM-SEP is the transpose of the one of MAS. However, it can be shown analytically that in NFM-SEP the large- N limits of the discrete sources' currents exist always, i.e., without restrictions such as (1) in MAS. Besides, in MAS the, discussed in Section II above, divergence of the normalized currents manifests itself as rapid, unnatural, exponentially large oscillations in the corresponding MAS currents. On the other hand, NFM-SEP does not give rise to such oscillations. Still, it is worth to emphasize that in both NFM-SEP and MAS, the fields generated by the discrete currents always converge to the true field, regardless of the positions of the auxiliary surfaces and of the occurrence (in MAS) or the nonoccurrence (in NFM-SEP) of oscillations. Further analysis on the issues of convergence/divergence of the discrete currents and the corresponding convergence of the fields as well as various numerical results for different 2-D scattering geometries are included in [12].

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