

Code-Verification Techniques for Electromagnetic Surface Integral Equations with Slot Models

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The accurate modeling of electromagnetic penetration is an important topic in computational electromagnetics. Electromagnetic penetration occurs through intentional or inadvertent openings in an otherwise closed electromagnetic scatterer, which prevent the contents from being fully shielded from external fields. To efficiently model electromagnetic penetration, aperture or slot models can be used with surface integral equations to solve Maxwell's equations.

Surface integral equations yield many code-verification challenges due to the various sources of numerical error and their possible interactions. These error sources arise from the use of basis functions to approximate the solution, quadrature to numerically integrate, and planar elements to represent curved surfaces.

For computational physics codes in general, it is necessary to establish the credibility of the simulation results. This is typically accomplished through verification and validation. Validation evaluates the appropriateness of the models instantiated in the code for representing the relevant physical phenomena, and is typically performed through comparison with experimental data. Verification, on the other hand, assesses the correctness of the numerical solutions produced by the code, through comparison with the expected theoretical behavior of the implemented numerical methods. Verification can be further divided into the activities of code verification and solution verification. Solution verification involves the estimation of the numerical error for a particular simulation, whereas code verification assesses the correctness of the numerical-method implementations.

Code verification is the focus of this work. In general, codes that approximately solve systems of differential, integral, or integro-differential equations can only be verified by using them to solve problems with known solutions. The discretization of the governing equations necessarily incurs some truncation error, and thus the approximate solutions produced from the discretized equations will incur an associated discretization error. If the solution to the problem is known, a measure of the discretization error may be evaluated directly from the approximate solution. The code may be verified by examining the rate at which the error decreases as the discretization is refined, thereby verifying the observed order of accuracy of the discretization scheme is the expected order of accuracy.

Exact solutions to systems of engineering interest are rare, and typically require dramatic simplifications to the problem. Hence, manufactured solutions are frequently employed to produce problems of sufficient complexity with known solutions. The method of manufactured solutions (MMS) is a general technique for constructing problems of arbitrary complexity with known solutions. Once a solution is manufactured, it is substituted directly into the governing equations to obtain a forcing term that coerces the solution to the manufactured solution.

Code verification has been performed on computational physics codes associated with several physics disciplines; however, existing literature contains few instances of MMS being used in the verification of software for integral equations. This is due to the simple fact that, while analytical differentiation is a straightforward exercise, analytical integration is not always possible. In this work, we provide approaches to separately measure the numerical errors arising from these different error sources for the method-of-moments implementation of the electric-field integral equation with a slot model. We demonstrate the effectiveness of these approaches for a variety of cases.