

Unifying Electrostatics and Magnetostatics Using Single Field-Impulse

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For more than 160 years, Maxwell equations in terms of electric ($\mathbf{E}$) and magnetic ($\mathbf{B}$) fields have been the core basis of electromagnetics (EM). The equations show the coupling of both fields in the case of electrodynamics, but they become separated into two sets independent of each other in the case of electrostatics and magnetostatics. Indeed, the electrostatics involves only curl-free equation and Gauss's law for electric field (or flux density), while the magnetostatics involves only Ampere's law and divergence-free equation for magnetic field (or flux density), cf. Figure 1: (1)-(2) and (3)-(4) respectively. These first-order field equations can also be replaced by Poisson's second-order potential ones, which are expressed in terms of electric scalar potential ϕ and magnetic vector potential $\mathbf{A}$ respectively. Although the potentials are of fundamental significance in quantum-EM interactions such as Aharonov-Bohm (AB) effect, the fields $\mathbf{E}$ and $\mathbf{B}$ are still favored as being more physical, causal, gauge-independent and measurable among EM community.

The two sets of electrostatics and magnetostatics equations involving two fields $\mathbf{E}$ and $\mathbf{B}$ (or two potentials ϕ and $\mathbf{A}$) have been in use without much change for over two centuries. There seems no (successful) attempt to unify them so far, perhaps not even by Maxwell who merely left his equations separated for statics. Here we present a single set of equations that unify both electrostatics and magnetostatics using a single physical quantity. They are based on our new fundamental equations of electromagnetics using field-impulses (E. L. Tan, "Fundamental equations of electromagnetics using field-impulses as physical field-integrators," IEEE Trans. Antennas Propag., vol. 70, no. 5, pp. 3450–3458, 2022). As shown in (5)-(6), our single set of curl and divergence equations involving single electric field-impulse $\mathbf{I}_E$ can unify both electrostatics and magnetostatics, by incorporating charge-integrator I_ρ and magnetic flux $\mathbf{B}$ as their specified sources. (Note that $\mathbf{B}$ can be practical sources themselves in magnetostatics, e.g. from magnets, magnetized coils, spin magnetization, etc.) Alternatively, the set of two first-order equations (5)-(6) can be converted into a single second-order equation (7), with usual current $\mathbf{J}$ as the source besides I_ρ (assuming homogeneous ϵ and μ). Moreover, the first- and/or second-order equations are readily extendable to electrodynamics by including extra $\frac{\partial}{\partial t}$ as in wave equation (8) using the same (but dynamic) sources. Therefore, (5)-(8) using $\mathbf{I}_E$ serve to reduce and replace Maxwell equations for complete unification of electromagnetics.

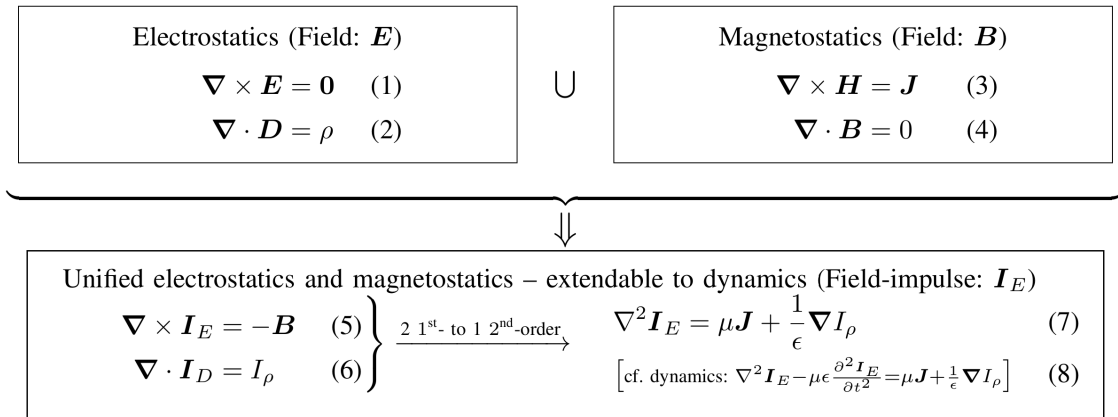


Figure 1: Unifying electrostatics and magnetostatics