

Finite-Energy Unidirectional Hopfion-Like Null Spatiotemporally Localized Electromagnetic Fields in Vacuum

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The basic (or pure) Hopfion is a finite-energy luminal spatiotemporally localized solution to Maxwell's equations in free space with unique topological properties. Specifically, all electric and magnetic field lines are closed loops, and any two electric (or magnetic) field lines are linked once with one another. However, the basic Hopfion has equally distributed forward (along the positive z -direction) and backward component

Using an appropriate factorization of the bidirectional splash mode, forward-propagating scalar-valued spatiotemporally localized waves of the general form $\psi(\vec{r}, t) = g^{-1}(\vec{r}, t)f(p, q)$, with $g(\vec{r}, t) = \sqrt{\rho^2 - c^2(t - it_s)}$, $p(\vec{r}, t) = g(\vec{r}, t) - i(z + iz)$ and $q(\vec{r}, t) = \rho \exp(-i\phi)[ic(t - it_s) + g(\vec{r}, t)]^{-1}$, t_s and z_x being free positive parameters, have been derived recently. Despite their unidirectionality, these scalar waves, as well as their vector-valued analogs, exhibit energy backflow.

It is the aim in this talk to present a general class of finite-energy unidirectional (along the positive z direction) Hopfion-like null spatiotemporally localized vector-valued electromagnetic waves in vacuum that are devoid of energy backflow. These structures are based on two Bateman conjugate functions: specifically, $\alpha(\vec{r}, t) = p(\vec{r}, t)^{-2}$ and $\beta(\vec{r}, t) = q(\vec{r}, t)$. Any functional of these two quantities obeys the nonlinear characteristic equation. Also, the constraint $\nabla \alpha \times \nabla \beta = -ic^{-1}[(\partial \alpha / \partial t) \nabla \beta - (\partial \beta / \partial t) \nabla \alpha]$. Two distinct methods are used to derive the solutions. The first one is conventional, involving a complex-valued scalar potential and a complex-valued vector potential satisfying the Lorenz gauge. This method is used to derive conservation laws for electromagnetic chirality and magnetic helicity. A second approach to the solutions involves two scalar vector potentials, one for the real electric field and the other for the real magnetic field. This approach, together with the first method, is used to derive a conservation law for the electric helicity.

The solutions have several of the topological characteristics associated with pure Hopfion. Linkages of the electric and magnetic field lines are similar but more complicated. The unidirectionality of the solutions is guaranteed by checking that the spectral component k_z assumes values only along the positive axis. The nullity property of the solutions is invariant under a Lorentz transformation. Also, it means that although the energy transport velocity (the ratio of the Poynting vector and the electromagnetic energy density) can vary with space and time, its modulus equals the speed of light in vacuum. For the solutions under consideration, the component of the energy transport velocity along the direction of propagation is independent of z . It is equal to c , the speed of light in vacuum, on axis ($\rho = 0$) and subluminal for all other values of the radial variable.