

An Analysis of Spherical Wire Antenna Winding Patterns: Quality Factors, Spherical Waves, and Current Densities

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The reduction of an antenna's electrical size most often coincides with increased restriction on other parameters, such as directivity, radiation efficiency, and impedance bandwidth (to name a few). Spherical waves have been instrumental to a rigorous understanding of these observations, namely, via a set of canonical fields and currents whereby derivative properties are computed and generally recognized to bound small antenna performance. The first such work was presented in Chu's seminal paper (L. J. Chu, "Physical limitations of omni-directional antennas," *J. Appl. Phys.*, vol. 19, no. 12, pp. 1163–1175, Dec. 1948), where the quality factor Q of the spherical waves were extracted using rigorously derived equivalent circuits. For small antennas, Chu's circuits predict

$$Q \sim \mathcal{O}\left[1/(ka)^3\right], \quad (1)$$

a result that has been independently reproduced by many authors. Here, ka is the electrical size as measured by product of the resonant wavenumber k , and circumscribing sphere radius a , of the antenna. Equation (1) then suggests that the fractional bandwidth decreases sharply with decreasing electrical size.

This spherical wave paradigm has caught the attention of antenna designers as inspiration for the synthesis of small antennas at the bounds of what is considered to be physically possible under the assumptions of the aforementioned analysis (which include—but are not limited to—passivity, linearity, and time-invariance). These antenna designs and spherical wave-based theories are co-validating exercises. Both methods have generally converged onto the idea that the best performing antenna geometry is that where the radiating currents flow predominantly near the surface of a sphere. A recent article by Murray and Iyer (A. B. Murray and A. K. Iyer, "An exact synthesis procedure for minimum quality factor spherical wire antennas," *IEEE Trans. Antennas Propag.*, in press) has demonstrated a framework by which the spherical waves can be used to directly synthesize an antenna geometry, and vice versa, for low-order spherical waves and electrically small antennas. The candidate geometry used by Murray and Iyer for arbitrary control over the TM_{10} and TE_{10} spherical wave amplitudes is a spherical wire antenna (SWA) of radius a , with parametrization

$$(r(t), \theta(t), \phi(t)) = (a, t, N \ln |\tan(t/2)| + \phi_0), \quad t \in [0, \pi], \quad (2)$$

where N is the ratio of current entering the TE_{10} equivalent circuit to that entering the TM_{10} circuit, and ϕ_0 is an arbitrary azimuthal offset which may be multivalued (in the case of multiple wires). This geometry was shown in full-wave simulation to have a near-perfect correspondence with its theoretically prescribed spherical waves (hence Q and far fields), when the assumptions on the wires' current profiles were met.

In this presentation, we will analyze the performance of this SWA parametrization against some perhaps more obvious choices, such as that parametric curve which assumes constant pitch:

$$(r(t), \theta(t), \phi(t)) = (a, t, pt + \phi_0), \quad t \in [0, \pi] \quad (3)$$

(for some pitch parameter p). The spherical waves and current patterns between the SWAs will be quantified and used to understand the effect of wire parametrization on antenna Q . We hope this discussion aids in advising designers interested in incorporating spherical wires into their antennas with the aim of improving antenna bandwidth, radiation pattern control, or otherwise facilitating the use of spherical waves in antenna design.