

Extraction Methods of the Dominant Delay in Simulation of Electromagnetic Waves in the Low Troposphere

Hang Zhou⁽¹⁾ and Alexandre Chabory⁽²⁾

⁽¹⁾ Sino-European Institute of Aviation Engineering, Civil Aviation University of China, Tianjin, China (h-zhou@cauc.edu.cn)

⁽²⁾ ENAC Lab, Université de Toulouse, Toulouse, France

Abstract—This paper presents methods for extracting the dominant delay in electromagnetic wave simulations in the low troposphere. Extracting the dominant delay is critical notably for systems where the delay is an important parameter. The study evaluates the limitations of group delay determination, offering detailed insights into their applicability and performance under various scenarios. Additionally, the paper provides specific guidelines for selecting optimal frequency increments to avoid phase ambiguity, ensuring more accurate and reliable signal analysis in complex environments.

I. INTRODUCTION

Modeling long-distance wave propagation is a challenge with wide-ranging applications. The parabolic equation (PE) method is used to approximate the wave equation, enabling the simulation of electromagnetic wave propagation in complex environments. To solve the PE, frequency-domain techniques such as the split-step Fourier method (SSF) are widely employed, particularly for scenarios involving the lower troposphere [1]. As an alternative, the split-step wavelet method (SSW) has been developed to reduce computational complexity while maintaining accuracy [2]. These methods are effective in handling inhomogeneous refractive indices, ground interactions, and complex topographies [3].

Once the electromagnetic wave propagation has been simulated using the methods outlined above, extracting the dominant delay is important for analysis of wave propagation. This paper introduces distinct methods for extracting the dominant delay. Section II presents the signal model. Sections III, IV, and V present the dominant delay extraction methods from the phase of the complete field, from the phase derivative of the complete field, and from the phase derivative of the reduced field, respectively.

II. SIGNAL MODEL

This paper focuses on horizontally polarized waves, as the approach for vertically polarized waves is similar. At any position, the electric field can be modeled as

$$E(f) = a_0 e^{j\phi_0} e^{-j2\pi f\tau_0} = a_0 e^{j\phi_0} e^{-jk_0 l_0}, \quad (1)$$

where a_0 is the amplitude, ϕ_0 the phase, τ_0 the delay, and l_0 the optical path length. In split-step simulations, such as SSF and SSW, at a frequency f_0 , both the magnitude a_0 and phase $\phi_0 - 2\pi f_0 \tau_0$ are directly computed. Methods for extracting group delay from results on the x - z computation grid are detailed in the following sections.

III. FROM THE PHASE OF THE COMPLETE FIELD

Determining delay from the phase of the complete field is feasible in non-dispersive cases, where atmospheric and ground properties are frequency-independent. If grid steps are smaller than $\lambda_0/2$ (λ_0 being the free-space wavelength at f_0), phase unwrapping can convert radians to delay.

However, in split-step methods, the grid step in x exceeds $\lambda_0/2$, making direct delay computation infeasible.

IV. FROM THE PHASE DERIVATIVE OF THE COMPLETE FIELD

A common approach to determine the delay of the dominant delay is through the group delay, applicable even in dispersive environments, defined as

$$\tau_0 = -\frac{1}{2\pi} \frac{d\angle E}{df}, \quad (2)$$

where $\angle$ denotes the phase angle of a complex number.

A straightforward approach to compute the group delay involves using finite differences. This requires simulations at frequencies: $f_0 - \delta f/2$ and $f_0 + \delta f/2$, with δf a small frequency increment. Using centered finite differences, the group delay can be approximated as

$$\tau_0 = -\frac{1}{2\pi} \frac{\angle E(f_0 + \delta f/2) - \angle E(f_0 - \delta f/2)}{\delta f}. \quad (3)$$

For better numerical performances, (3) can be rewritten as

$$\tau_0 = -\frac{1}{2\pi} \frac{\angle (E(f_0 + \delta f/2)/E(f_0 - \delta f/2))}{\delta f}. \quad (4)$$

The choice of δf is important. If δf is too large, phase ambiguity arises. Conversely, if δf is too small, the phase difference becomes smaller than the simulation's numerical error. Two limitations are discussed in detail below:

- 1) Phase ambiguity: If the phase difference between $f_0 \pm \delta f/2$ exceeds 2π , phase ambiguity arises. To avoid this, the following criteria should be satisfied

$$\tau_0 \leq \frac{c_0}{\delta f}. \quad (5)$$

This limitation is plotted in Figure 1a. The horizontal axis represents δf , while the vertical axis shows the maximum computable delay. With $\delta f = 1$ kHz, delays up to 300 km can be computed. This is a typical maximal

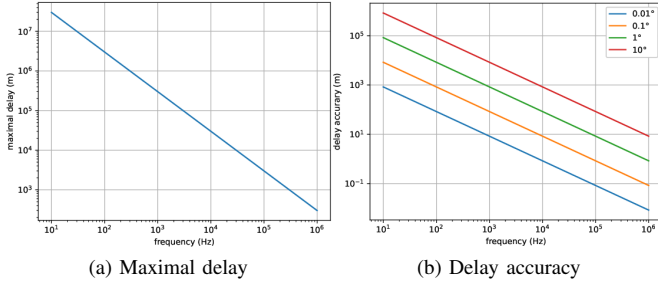


Fig. 1: Limitation of the group delay determination from the phase derivative of the complete field.

value in the troposphere, where propagation is limited by the radio horizon.

- 2) Phase accuracy: If the phase in (4) contains an error of ϵ_ϕ , the resulting error in the delay estimation is

$$\epsilon_\tau = \frac{\epsilon_\phi}{2\pi\delta f}. \quad (6)$$

This limitation is illustrated in Figure 1b. The vertical axis indicates the delay error, and the lines with different colors represent distinct phase errors. For example, when $\delta f = 1$ kHz, the simulation must deliver an accurate phase value (below 0.01°) to ensure reliable delay estimation.

In conclusion, finding an appropriate trade-off for δf in this method can be challenging.

V. FROM THE PHASE DERIVATIVE OF THE REDUCED FIELD

An alternative approach involves using the phase of the reduced field. Since the main phase evolution in x is removed, phase ambiguity should be mitigated. The reduced field is $E(x, z) = u(x, z)e^{-jk_0x}$, with k_0 the free-space wavenumber. Thus, the phase is $\angle E(x, z) = \angle u(x, z) - k_0x$ and the delay is expressed as

$$\tau_0 = -\frac{1}{2\pi} \frac{d\angle u}{df} + \frac{x}{c_0}. \quad (7)$$

Using centered finite differences, the delay can be obtained from two simulations at nearby frequencies

$$\tau_0 = -\frac{1}{2\pi\delta f} \angle \frac{u(f + \delta f/2)}{u(f - \delta f/2)} + \frac{x}{c_0}. \quad (8)$$

To test this method's ability to mitigate phase ambiguity, we apply it to the far-field radiation of a source at $x = 0, z = z_s$ in a homogeneous atmosphere with refractive index n . The reduced field phase for such radiation is

$$-kr + k_0x + A = -\frac{2\pi f}{c_0}(-nr + x) + A, \quad (9)$$

where $r = \sqrt{x^2 + (z - z_s)^2}$ and A is a constant.

Phase ambiguity in (8) does not occur if

$$|z - z_s| \leq \frac{1}{n} \sqrt{\left(x + \frac{c_0}{\delta f}\right)^2 - n^2x^2}. \quad (10)$$

The results are presented in Figure 2 for various refractivity defined from $n = 1 + N \cdot 10^{-6}$, along with frequency increments δf . The areas enclosed by the different colored curves indicate regions where phase ambiguity does not occur for various δf values. As δf decreases, phase ambiguity is avoided over a larger area, which also depends on the value of N . However, a smaller δf requires the simulation method to have higher precision in distinguishing results for closely spaced frequencies. Therefore, selecting an appropriate δf involves balancing these two factors.

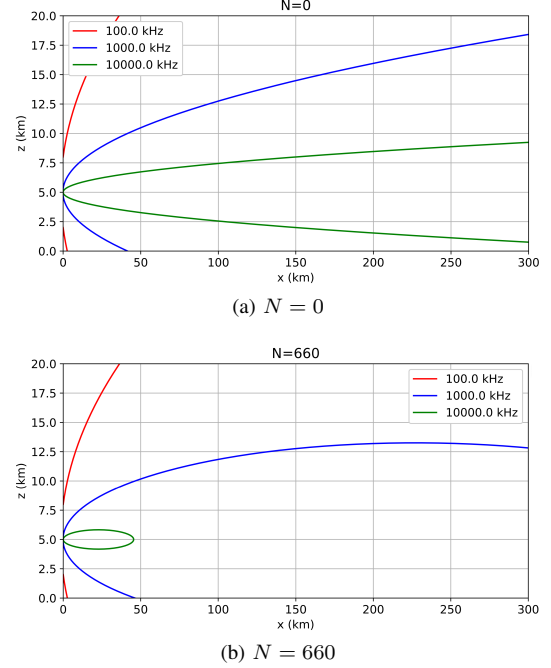


Fig. 2: Limitation of the group delay determination from the phase derivative of the reduced field.

VI. CONCLUSION

This paper has introduced distinct methods for extracting the dominant delay in electromagnetic wave simulations in the low troposphere. By evaluating the limitations of group delay determination, this paper provides guidance on selecting appropriate frequency steps to avoid phase ambiguity.

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REFERENCES

- [1] M. Levy, *Parabolic Equation Methods for Electromagnetic Wave Propagation*. Edison, NJ, USA: IET, 2000.
- [2] H. Zhou, R. Douvenot, and A. Chabory, "Modeling the long-range wave propagation by a split-step wavelet method", *J. Comput. Phys.*, vol. 402, Feb. 2020.
- [3] T. Bonnafont, R. Douvenot, and A. Chabory, "A local split-step wavelet method for the long range propagation simulation in 2D", *Radio Sci.*, vol. 56, no. 2, pp. 1–11, Feb. 2021.